New Consensus Protocols for Agent Networks with Discrete Time Dynamics and Distributed Task Assignment

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Outline

- The Consensus Problem
- The Consensus Algorithms
- A New Class of Consensus Algorithms
- Convergence Properties of the Iterative Schemes
- Convergence Performance Analysis
- Conclusions and Future Research

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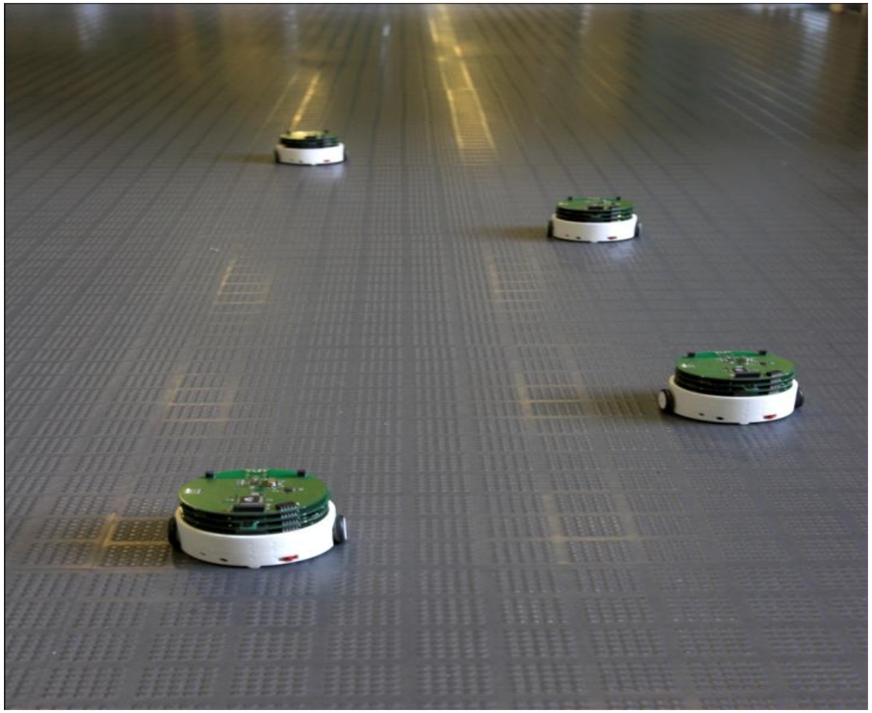
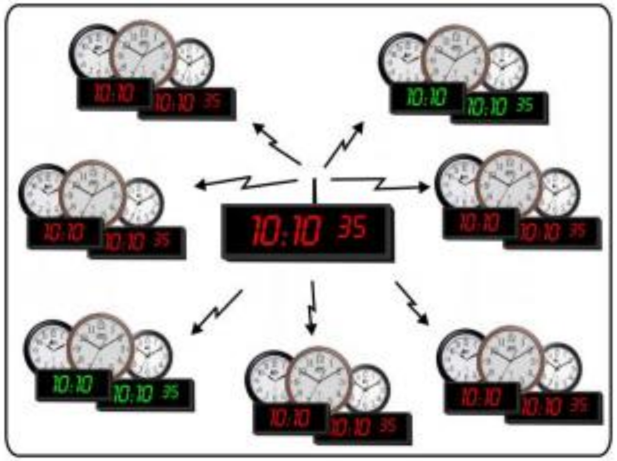
The Consensus problem

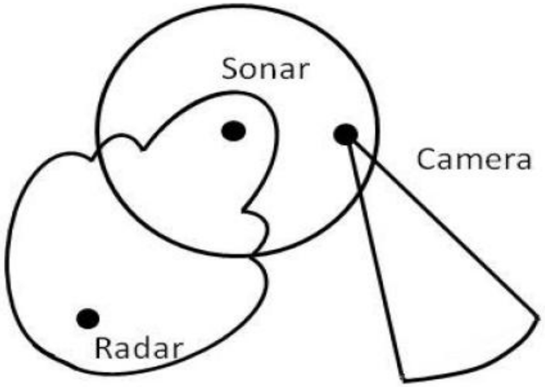
Consensus deals with the problem of distributed coordination of networks of dynamic agents.

Typically, agents are intelligent sensors and processors that try to reach agreement on a common value or decision by exchanging tentative values and combining them.



The Consensus problem





- Consensus paradigm is applied to different fields (cooperation, synchronization, flocking, load balancing):
  - cooperative control of unmanned air vehicles,
  - mobile robots,
  - autonomous underwater vehicles,
  - satellites, aircraft, spacecraft
  - automated highway systems.

The Consensus problem

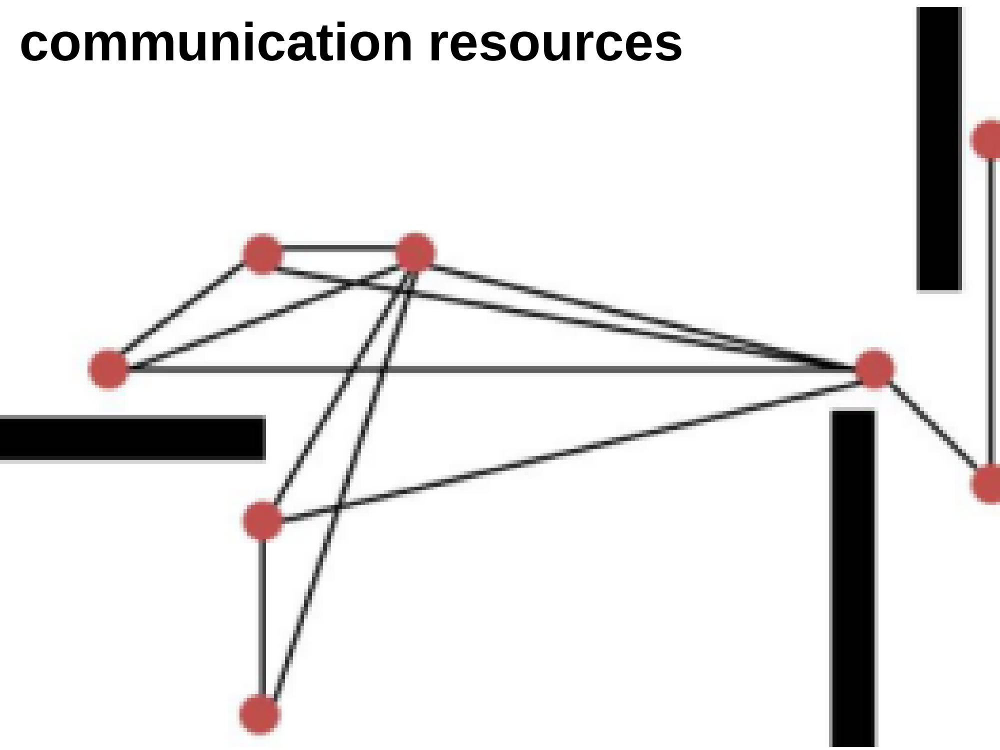
Sensor networks and sensor swarms are an exciting frontier of research. In addition to hardware advances needed in **miniaturization**, **communication** and **powering individual sensors**, the **WAY** in which the data are collected and **communicated**.

A much more revolutionary idea is the concept of **Sensor Swarming**, where the swarm itself exhibits 'emergent behavior' or 'intelligence'.

If is necessary to develop and simulate **a high performing sensor coordination protocol.**

The Consensus problem

Obstacles = constraints in communication resources

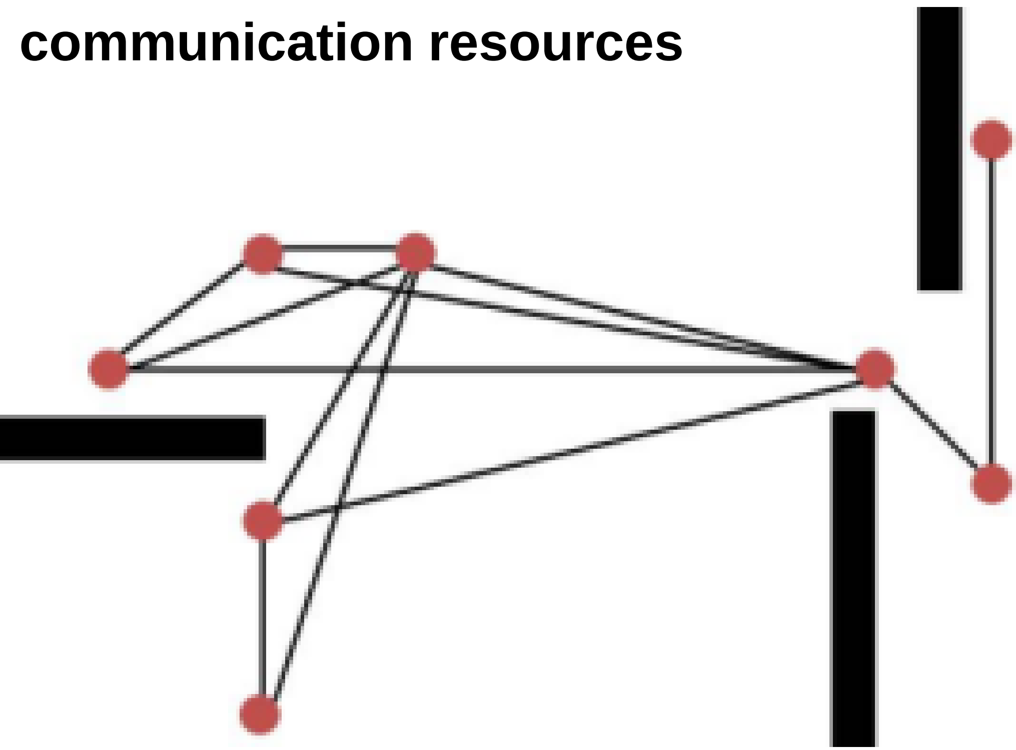


The fundamental paradigms in distributed decision and consensus:

- **(i)** **information** relevant for the solution of the problem is **distributed** all over a network of processors with limited memory and computation capability,
- (ii) the overall computation relies only on **local, distributed computation and information exchange** among processors,
- (iii) each agent can **communicate** with a small subset of neighbor agents.

The Consensus problem

Obstacles = constraints in communication resources

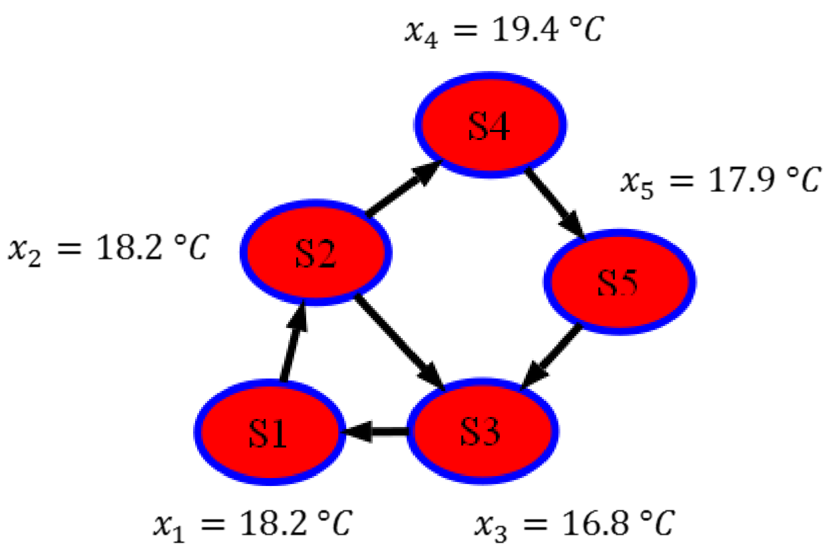


The main difficulty of the problem resides in the communication constraints.

The communication across the links can be assumed digital and possibly subject to bandwidth constraints, interferences, erasures, packet losses, noise, delays.

The Consensus problem

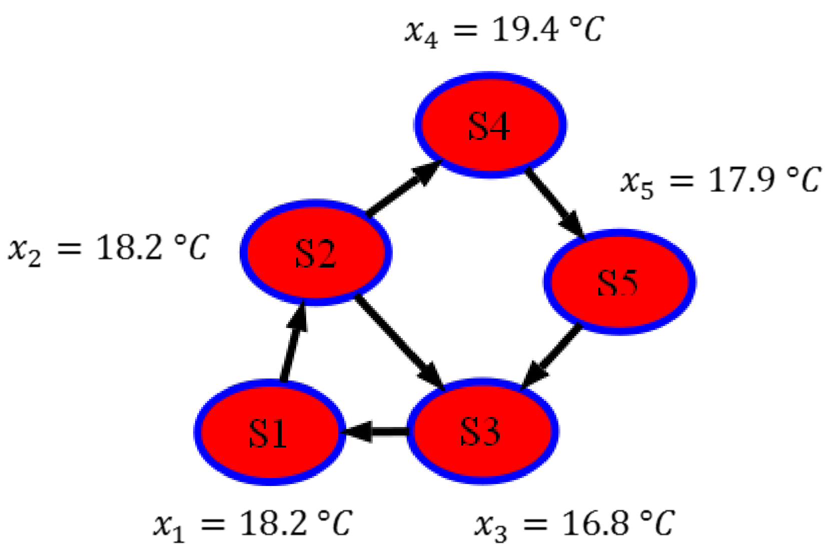
***A classic consensus example*: A temperature sensor network provides different values**



- The interaction topology of a network of agents is represented using a directed graph $G=(V,E)$:
  - $V=\{1,2,..,n\}$ set of nodes
  - $E \subseteq V \times V$ set of edges.
- $\boldsymbol{A}=[a_{ij}]$ is the adjacency matrix of the graph

The Consensus problem

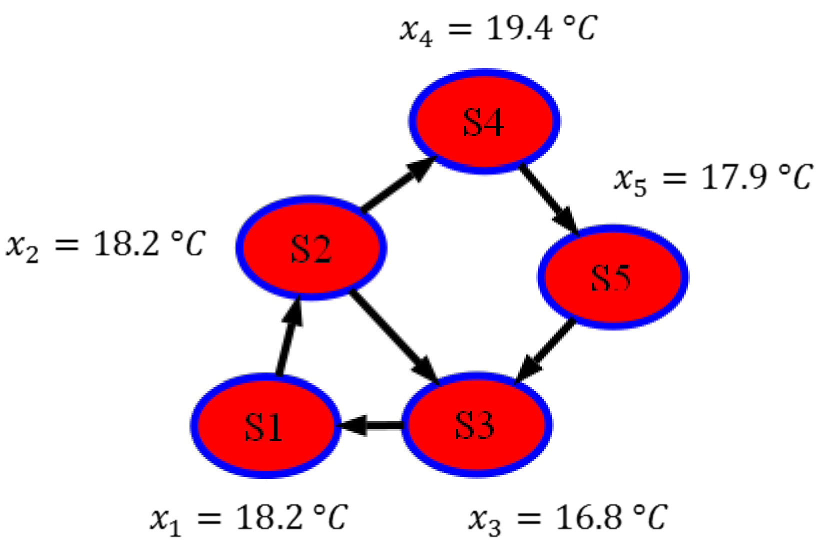
***Example*: A temperature sensor network provides different values**



- x_i is the state of agent *i*,
- i.e. the measure provided by each sensor
- The agents exchange and combine their values with the neighbor agents.
- We say that the nodes of a network have reached a consensus if $x_i = x_j$ for all $i,j \in V$.

The Consensus problem

***Example*: A temperature sensor network provides different values**



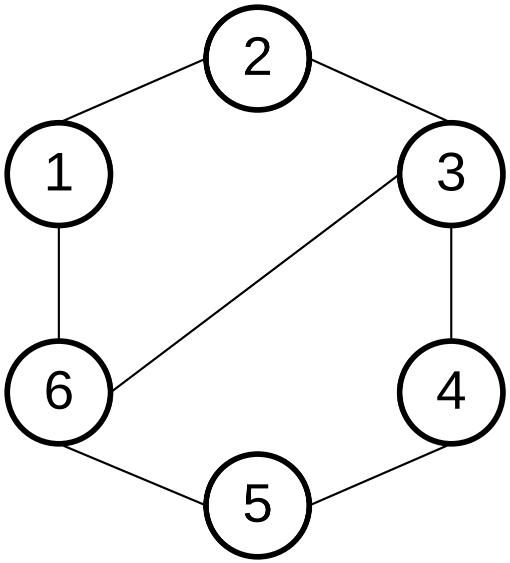
For example in this case the consensus is reached if:

$$x_1 = x_2 = x_3 = x_4 = x_5 = 18.1\,°C =$$

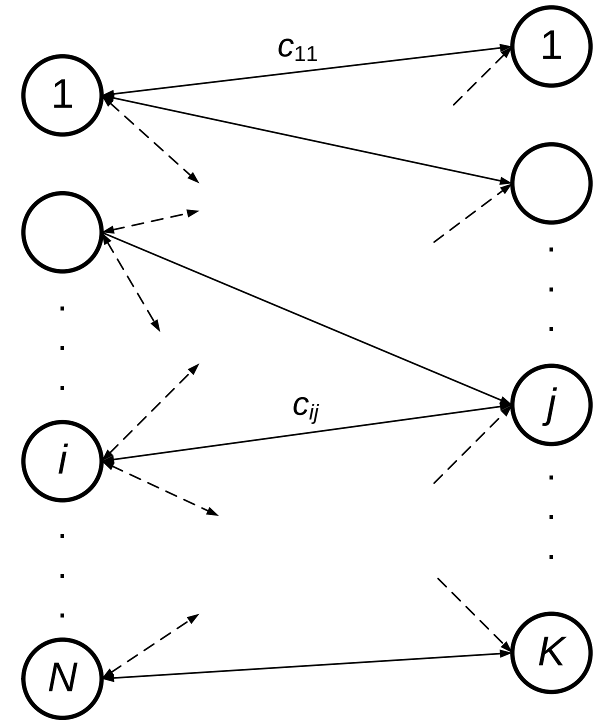
$$= \frac{x_1(0) + x_2(0) + x_3(0) + x_4(0) + x_5(0)}{5}$$

The Consensus problem

A related consensus problem: task assignment problem → quantized consensus



Agents

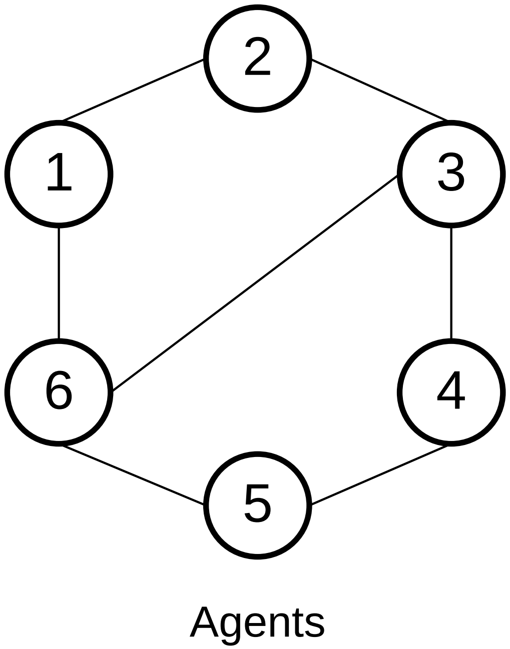


Agents Tasks

- The requirements are:
- i) assigning all the tasks to the agents;
- ii) assigning to each agent no more than *M* tasks;
- iii) minimizing the maximum total load of each agent.

The Consensus problem

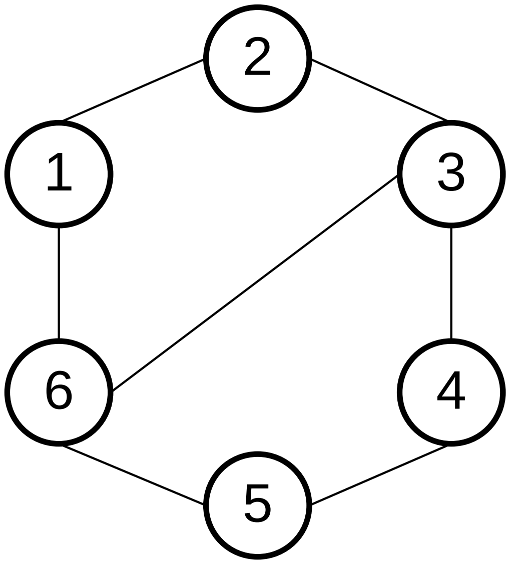
A related consensus problem:
task assignment problem → quantized consensus



- Among the many algorithms for quantized consensus particularly interesting is the so called **gossip algorithm**:
- at every time instant a randomly chosen pair of agents communicates and optimizes in a decentralized approach the task distribution.

The Consensus problem

A related consensus problem:
task assignment problem → quantized consensus



Agents

The group of agents can negotiate about an optimal distribution of the tasks.

Solutions of this problem can be obtained by:

- game-theoretic negotiation mechanisms,
- dynamic reassignment problems,
- minimum-time assignment problems for robotic networks
- quantized gossip algorithms
- distributed optimization strategies

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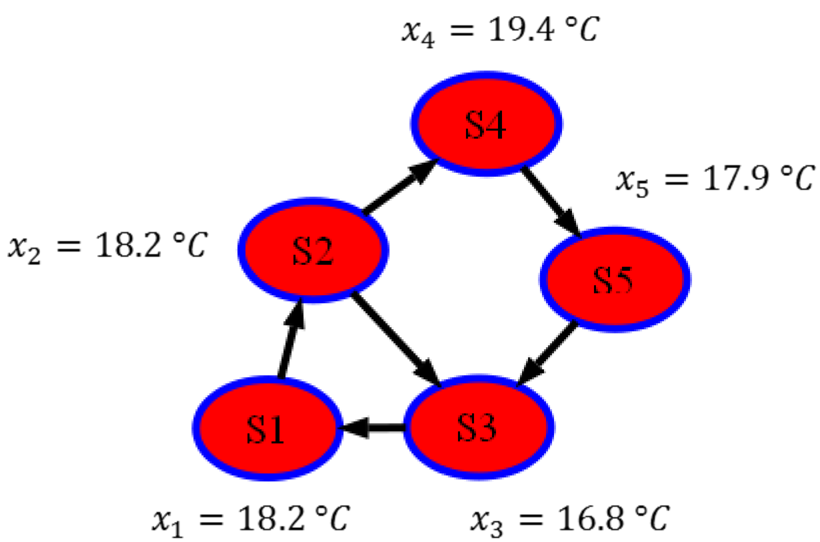
The Consensus Algorithms

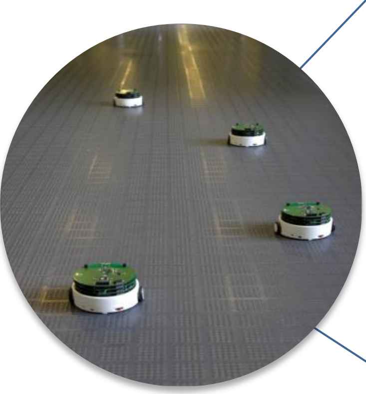
***Example*: A temperature sensor network provides different values**



We focus on consensus algorithms for agent networks where the node states are described by real values.

- A consensus algorithm is an **interaction rule** that specifies the information exchange between an agent and all of its neighbors on the network.

The Consensus Algorithms



Continuous time model

$$\dot{x}_i(t) = \sum\nolimits_{j \in N_i} a_{ij}(x_j(t) - x_i(t)) \;\; i = 1, 2, \ldots n$$

Discrete time model

$$x_i(t+1) = x_i(t) + \varepsilon \sum_{j \in N_i} a_{ij}(x_j(t) - x_i(t)),\, t \in \mathbb{N}$$

The Consensus Algorithms

- A well-known consensus algorithm that solves the agreement problem in a network of agents with discrete time model is:

$$x_i(k+1) = x_i(k) + u_i(k)$$

- with

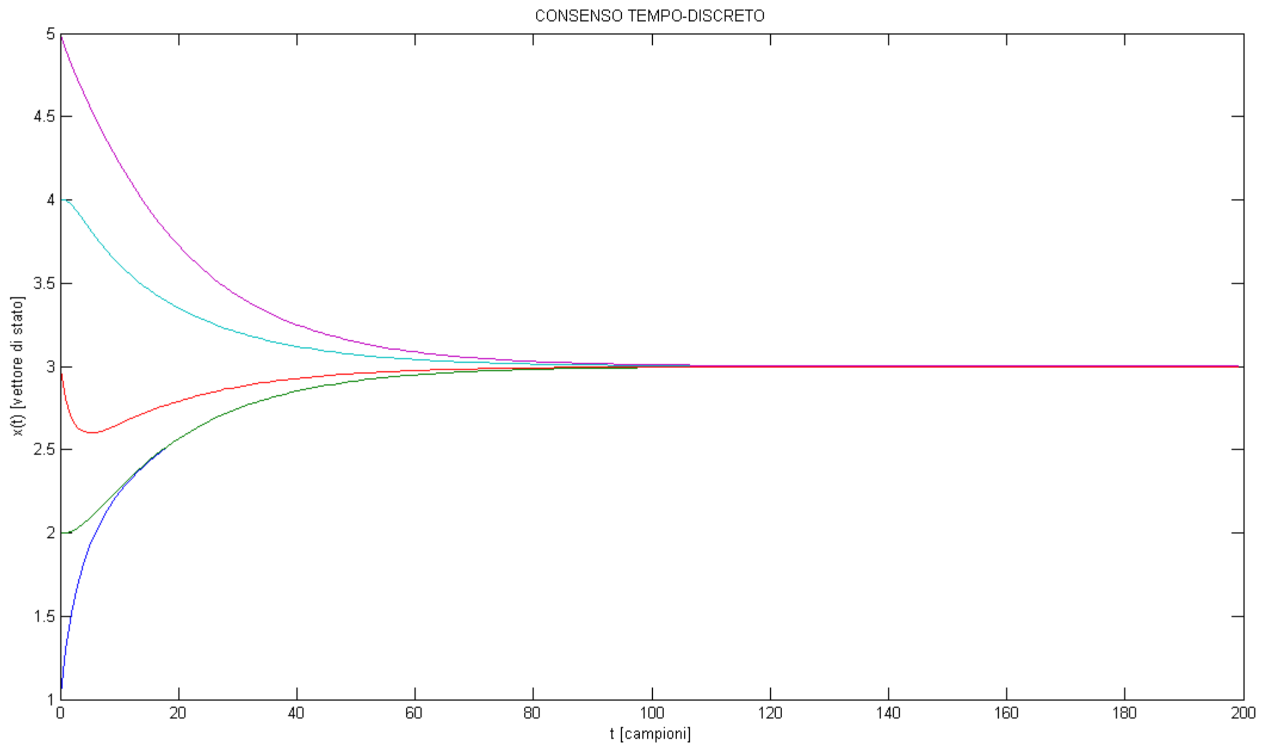
$$u_i(k) = \varepsilon \sum_{j \in N_i} a_{ij}(x_j(k) - x_i(k))$$

where ε is the step size

The Consensus Algorithms

- Hence the algorithm is:

$$x_i(k+1) = x_i(k) + \varepsilon \sum_{j \in N_i} a_{ij}(x_j(k) - x_i(k))$$



The Consensus Algorithms

- The consensus convergence properties are related to the non–negative matrices and Markov Chain theory.

- The iterative scheme can be written as:

$$x(k+1) = P_{\mathcal{E}} x(k)$$

The Consensus algorithms

- $P_\varepsilon = (I - \varepsilon L)$ is the iteration matrix, ε is the step-size parameter, I is the identity matrix and L is the *graph Laplacian* induced by the graph G and defined as:

$$l_{ij} = \begin{cases} \sum_{k=1, k \neq j}^{n} a_{ij} & \text{if } j = i \\ -a_{ij} & \text{if } j \neq i \end{cases}$$

- Denoting by Δ the maximum node out-degree of graph G, P_ε is a nonnegative and stochastic matrix for all $\varepsilon \in (0, 1/\Delta)$.

The Consensus Algorithms

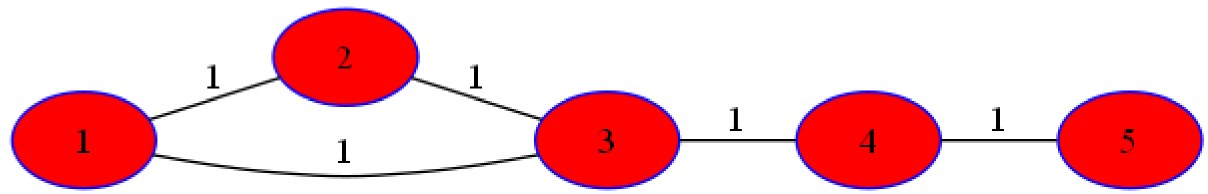
- The convergence analysis of the discrete-time consensus algorithm relies on the following well-known lemma in matrix theory (Perron-Frobenius):
- *Lemma 1:* Let B be a primitive (an irreducible stochastic acyclic matrix with only one eigenvalue λ=1) with left and right eigenvectors w and v, respectively, satisfying $Bv=v$, $w^T B=w^T$, and $v^T w=1$. Then:

$$\lim_{k\to\infty} B^k = vw^T.$$

The Consensus algorithms

Non oriented strongly connected graph

G_1



$$\mathbf{A}_1 = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Adjacency matrix

$$\mathbf{\Delta}_1 = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Node outdegree matrix

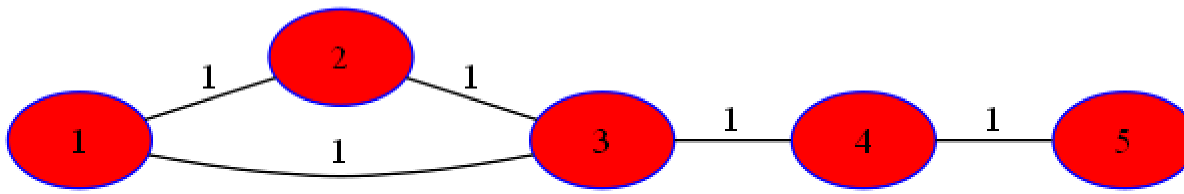
$$\mathbf{L}_1 = \begin{pmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ -1 & -1 & 3 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

Laplacian matrix

The Consensus algorithms

Non oriented strongly connected graph

G_1



$$\varepsilon = 0.3(1/d_{max}(G_1)) = \frac{0.3}{3} = 0.1$$

$$\mathbf{P}_1 = \mathbf{I} - 0.1\mathbf{L}_1$$

$$\mathbf{P}_1 = \begin{pmatrix} 0.8 & 0.1 & 0.1 & 0 & 0 \\ 0.1 & 0.8 & 0.1 & 0 & 0 \\ 0.1 & 0.1 & 0.7 & 0.1 & 0 \\ 0 & 0 & 0.1 & 0.8 & 0.1 \\ 0 & 0 & 0 & 0.1 & 0.9 \end{pmatrix}$$

Perron matrix

- P_1 is non negative, irreducible and aperiodic: **primitive.**
- Since the graph is balanced then **1 is the letf and right eigenvector** of P_1 .

The Consensus algorithms

$$lim_{k\to+\infty}\mathbf{P}_1\mathbf{x}(k) = lim_{k\to+\infty}\mathbf{P}_1^k\mathbf{x}(0) = \mathbf{v}_r\mathbf{v}_l^T\mathbf{x}(0)$$

$\mathbf{v}_r = a\mathbf{1},\ a \in \mathbb{R},\ a \neq 0$

$\mathbf{v}_l^T\mathbf{v}_r = 1$

$\mathbf{v}_l = b\mathbf{1},\ b \in \mathbb{R},\ b \neq 0$

$a = 1$:

$$\mathbf{v}_l = 0.2\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

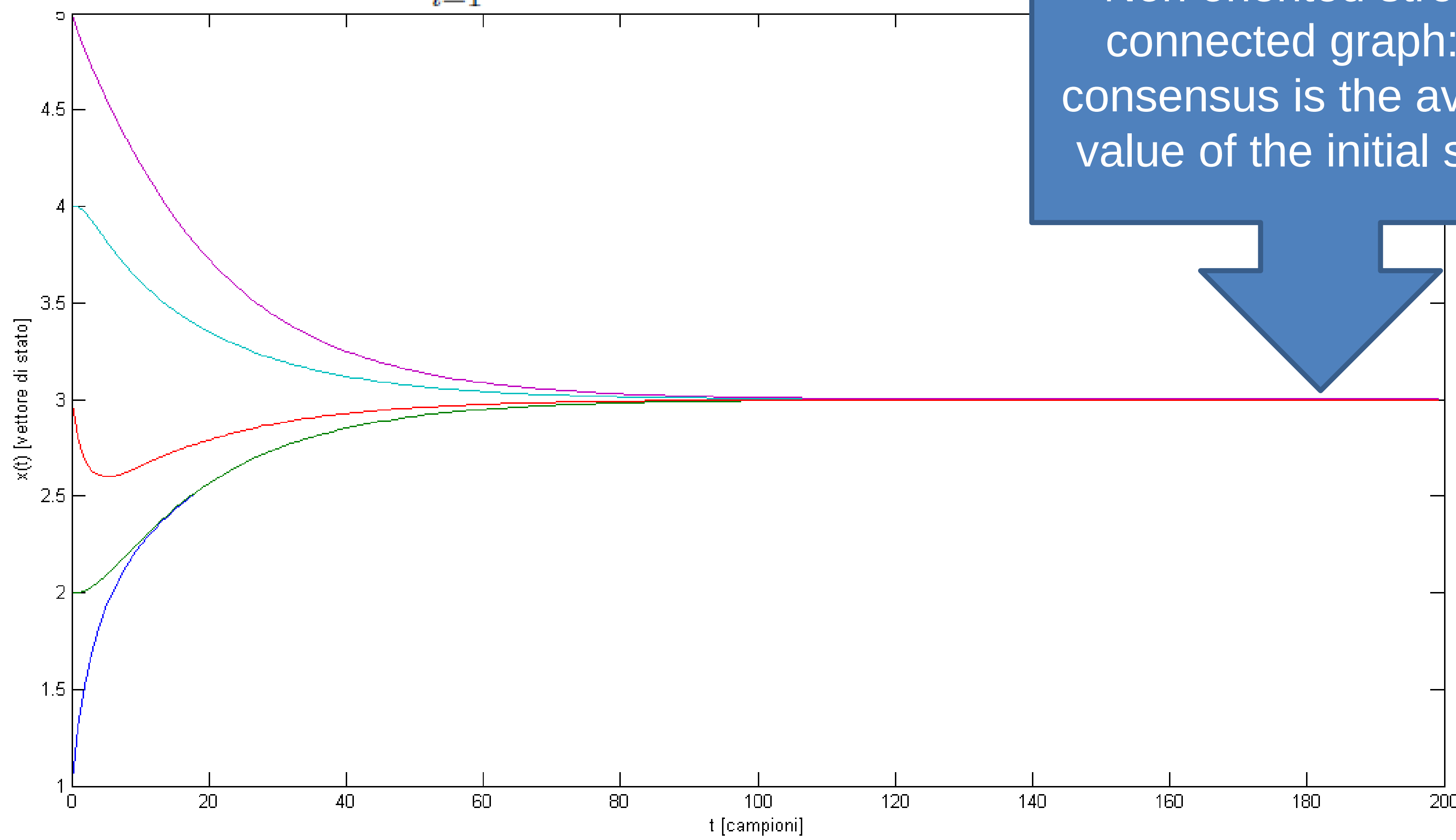
$$\alpha = 0.2\mathbf{1}^T\mathbf{x}(0) = 0.2\sum_{i=1}^{5} x_i(0) = 3$$

The Consensus algorithms

$$lim_{k\rightarrow+\infty}\mathbf{P}_1\mathbf{x}(k) = lim_{k\rightarrow+\infty}\mathbf{P}_1^k\mathbf{x}(0) = \mathbf{v}_r\mathbf{v}_l^T\mathbf{x}(0)$$

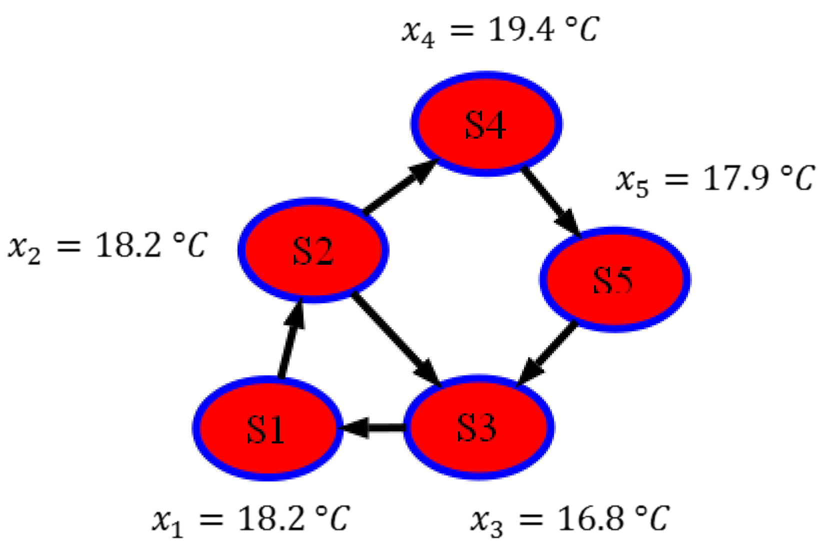
$$\alpha = 0.2\mathbf{1}^T\mathbf{x}(0) = 0.2\sum_{i=1}^{5} x_i(0) = 3$$

Non oriented strongly connected graph: the consensus is the average value of the initial states



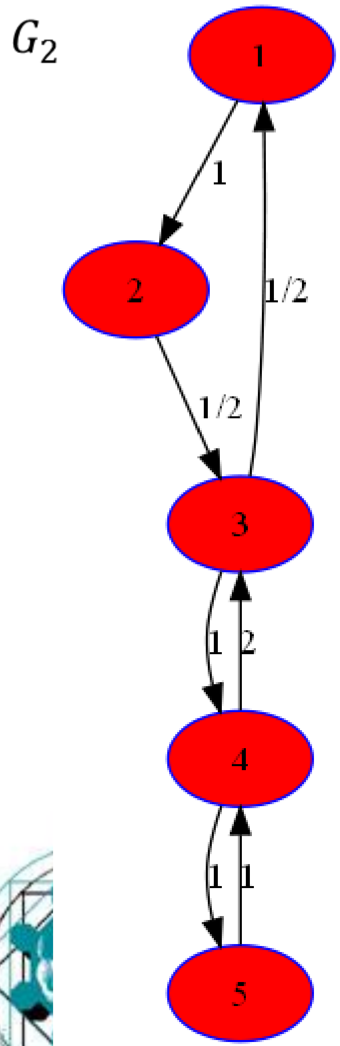
The Consensus algorithms

Theorem 1: (Olfati-Saber,fax, Murray, 2007)



- Let *G* be a strongly connected graph.
- Then a consensus is asymptotically reached for all the initial states;
- the group decision value is
- $x^* = \sum_i w_i x_i(0)$ with $\sum_i w_i = 1$
- and *w* is the left eigenvector of *P*ε.

The Consensus algorithms



Oriented strongly connected graph

$\varepsilon = 0.1$

$$P_2 = \begin{pmatrix} 0.9 & 0.1 & 0 & 0 & 0 \\ 0 & 0.95 & 0.05 & 0 & 0 \\ 0.05 & 0 & 0.85 & 0.1 & 0 \\ 0 & 0 & 0.2 & 0.7 & 0.1 \\ 0 & 0 & 0 & 0.1 & 0.9 \end{pmatrix}$$

- **P_2** is non negative, irreducible and aperiodic: primitive.
- **1 is the right eigenvector of the eigenvalue** 1.
- **w is the left eigenvector of the eigenvalue** 1.

The Consensus algorithms

The left eigenvalue of $\mathbf{P}_2$ is:

$$a\begin{pmatrix}1\\2\\2\\1\\1\end{pmatrix},\ a\in\mathbb{R},\ a\neq 0$$

Imposing the sum of the entries equal to 1:

$$\mathbf{v}_l=\begin{pmatrix}1/7\\2/7\\2/7\\1/7\\1/7\end{pmatrix},\ a\in\mathbb{R},\ a\neq 0$$

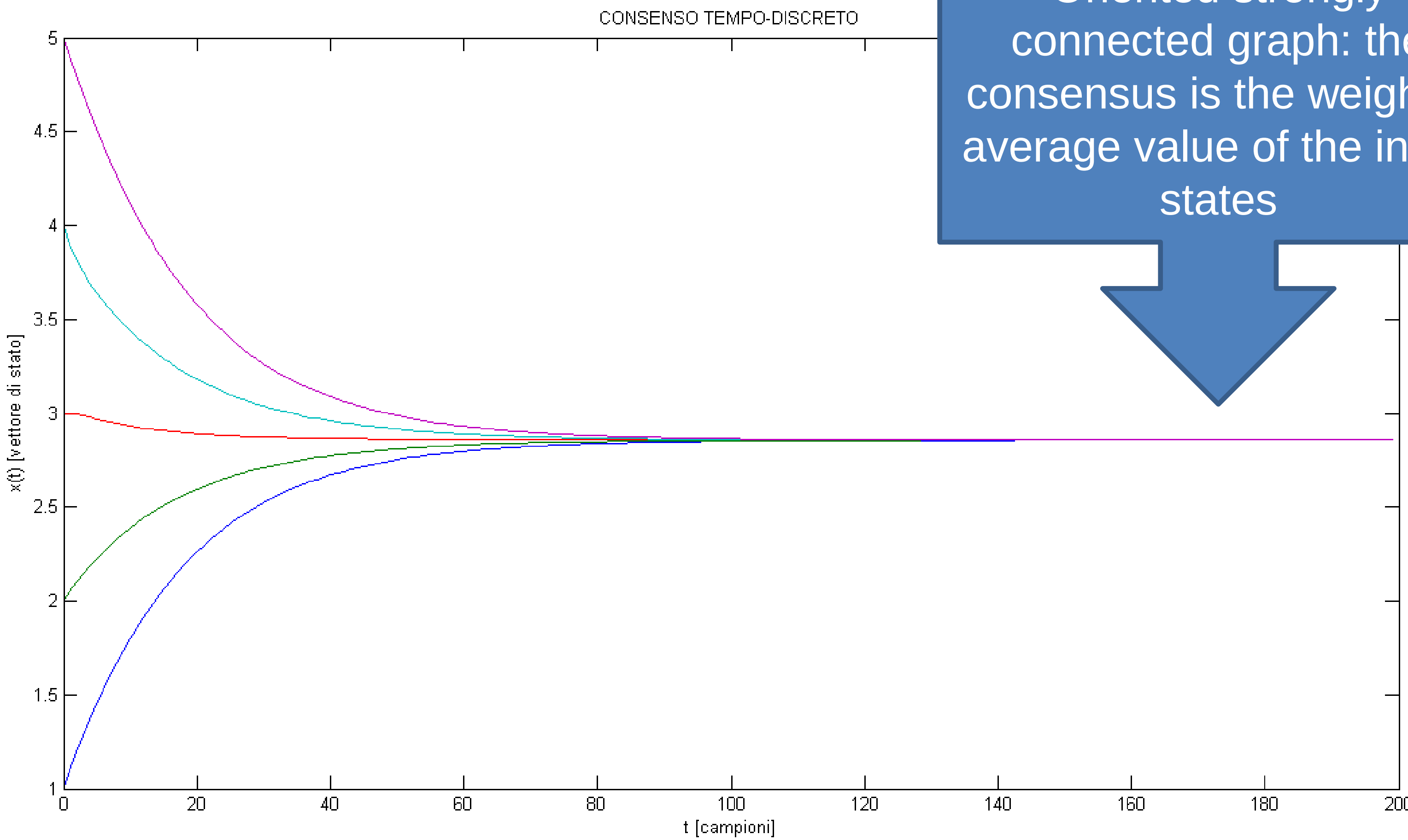
The consensus is:

$$\alpha=\frac{1}{7}x_1(0)+\frac{2}{7}x_2(0)+\frac{2}{7}x_3(0)+\frac{1}{7}x_4(0)+\frac{1}{7}x_5(0)=\frac{1+4+6+4+5}{7}=\frac{20}{7}\simeq 2.86\neq 3$$

The Consensus algorithms

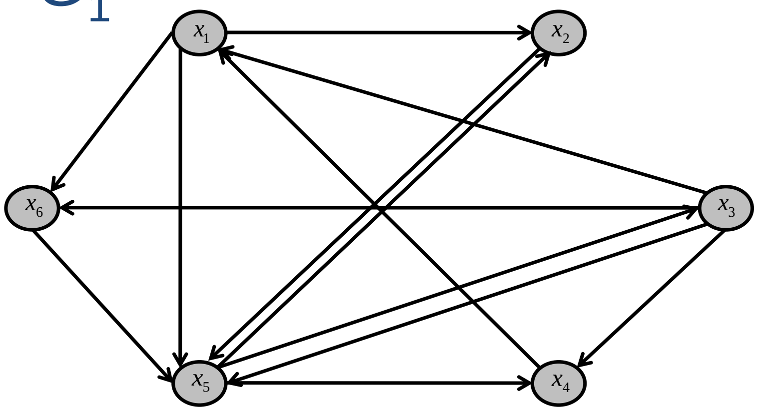
$$\alpha = \frac{1}{7}x_1(0) + \frac{2}{7}x_2(0) + \frac{2}{7}x_3(0) + \frac{1}{7}x_4(0) + \frac{1}{7}x_5(0) = \frac{1+4+6+4+5}{7} = \frac{20}{7} \simeq 2.86 \neq 3$$

Oriented strongly connected graph: the consensus is the weighed average value of the initial states

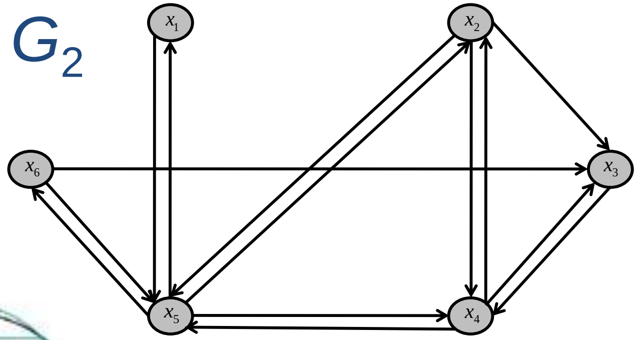


The drawbacks of the classic consensus algorithms

G_1



G_2



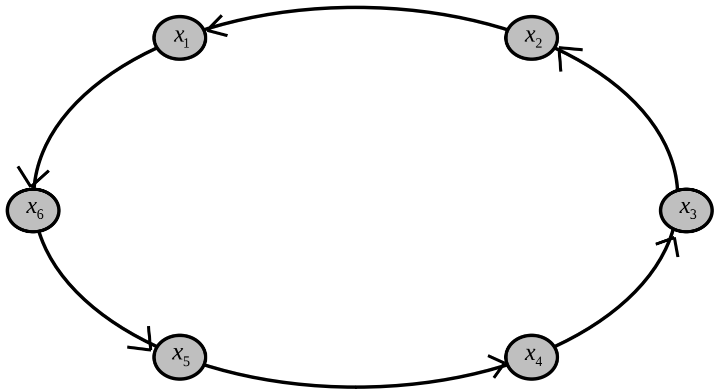
- G is strongly connected and aperiodic:
  - **The convergence is affected by the value** of ε.

TABLE I
CONVERGENCE PROPERTIES OF THE CONSENSUS ALGORITHMS

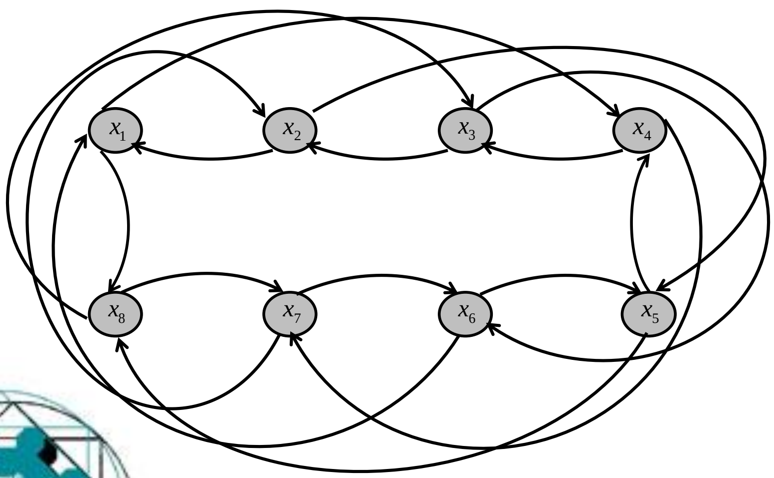
Consensus algorithm		G_1	G_2	G_3	G_4
	k^*	7	9	18	6
P_ε ε=0.5/Δ	x^*	0.718	0.726	0.703	0.716
	λ_2	0.904	0.875	0.866	0.707
	k^*	5	7	36	12
P_ε ε=0.8/Δ	x^*	0.718	0.726	0.703	0.716
	λ_2	0.847	0.800	0.916	0.825

The drawbacks of the classic consensus algorithms

G_3



G_4



- *G* is strongly connected and periodic:
  - The convergence is affected by the value of ε.

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A New Class of Consensus Algorithms

- The class of consensus algorithm is based on the *triangular splitting* of matrix $P\varepsilon = R+S$

- $Q(\varepsilon)=\{R, S \mid R\neq 0$ with $r_{ii}\neq 1$ and $r_{ii}\neq 0$ for $i=1,..n$ is a lower triangular matrix, $S\neq 0$ is an upper non negative triangular matrix, $R+S = P\varepsilon\}$.

- V. Boschian, M. P. Fanti, A.M. Mangini, W. Ukovich, “New Consensus Algorithms Based on a Positive Splitting Approach” **IEEE Conference on Decision and Control,** Orlando USA, December 12-16, 2011.
- M. P. Fanti, A.M. Mangini, W. Ukovich, V. Boschian, “New Consensus Protocols for Networks with Discrete Time Dynamics”, **American Control Conference,** Montreal, Canada, June 27-29, 2012.

A New Class of Consensus Algorithms

- The following lemma is proved:
- Consider $(R, S) \in Q(\varepsilon)$, then matrix $(I-R)^{-1}$ exists and is non-negative.

- Then each splitting induces the following iterative scheme.

$$x(k+1) = Rx(k+1) + Sx(k)$$

$$x(k+1) = (I-R)^{-1} Sx(k)$$

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Convergence Properties of the Iterative Schemes

- The following theorem guarantees the convergence of the algorithm that is induced by a triangular splitting:
- **Main Theorem:** Let P_ε be a stochastic irreducible matrix and w the left eigenvector of P_ε associated with the eigenvalue λ=1. Consider (R, S) $\in Q(\varepsilon)$ and assume that ***S* has no zero columns**. If there exists μ>0 such that $w^T S = \mu w^T$ i.e., ***w* is the left eigenvector of *S* for an eigenvalue μ>0,** then the induced algorithm converges for all the initial states and the group decision value is $x^*= vw^T x(0)$

Convergence Properties of the Iterative Schemes

The **proof scheme**: let consider the consensus iterative matrix

$$x(k+1) = (I-R)^{-1} S x(k) \qquad \Gamma = (I-R)^{-1} S$$

We show that:

- Γ is **stochastic**,
- **λ=1 is a simple eigenvalue** of Γ
- If S has no zero columns, then Γ is **irreducible and acyclic**

$\Rightarrow$ **Γ is primitive**

If there exists μ>0 such that $w^T S = \mu w^T$ then the iterative scheme converges to the same group decision value.

Characterization of the Iterative Schemes

- The proposed consensus algorithm is implemented by the following iterative scheme:

$$(1-r_{ii})x_i(k+1)=\sum_{j=1}^{i-1} p_{\varepsilon ij}x_j(k+1)+s_{ii}x_i(k)+\sum_{j=i+1}^{n} p_{\varepsilon ij}x_j(k)$$

- The iterative algorithm establishes an order to update the values of each agent state.

- To update the state at the time k+1, agent i-th uses the already determined values of the states for j=1,…,i-1.

Characterization of the Iterative Schemes

- In order to obtain an triangular splitting $(R,S)\in Q(\varepsilon)$ such that S satisfies the conditions of the convergence Theorem, the following set of linear constraints is defined:

$$\varphi(P_\varepsilon,w)=\begin{cases}\mu>0\\ \sum_{j=1}^{i} w_j s_{ji}-w_i\mu=0 \text{ for } i=1,\dots,n\\ s_{ii}\geq 0 \text{ for } i=1,\dots,n\\ s_{ij}=0 \text{ for } i>j,\ i,j=1,\dots,n\\ s_{ij}=p_{\varepsilon ij} \text{ for } i<j,\ i,j=1,\dots,n\\ \mathbf{1}^{\mathrm{T}}S>0\end{cases} \qquad (9)$$

Characterization of the Iterative Schemes

We prove that there exists a triangular splitting $(R,S) \in Q(\varepsilon)$ that satisfies the set of constraints:

$$s_{ij} = 0 \text{ for } i > j,\ s_{ij} = p_{\varepsilon ij} \text{ for } i < j \text{ with } i, j = 1, \dots, n$$

$$s_{11} = \mu = \sum_{j=1}^{i^*-1} \frac{w_j}{w_{i^*}} s_{ji^*} \quad s_{ii} = \sum_{j=1}^{i^*-1} \frac{w_j}{w_{i^*}} s_{ji^*} - \sum_{j=1}^{i-1} \frac{w_j}{w_i} s_{ji} \text{ for } i = 2, \dots, n$$

- We prove that in this case the obtained matrix

$$\Gamma = (I - R)^{-1} S$$

is independent from ε.

Characterization of the Iterative Schemes

$$\alpha_1 = L_{i*} = \eta \sum_{j=1}^{i*-1} \frac{w_j}{w_{i*}} a_{ji*}$$

$$\alpha_i = \eta L_{i*} - L_i = \eta \sum_{j=1}^{i*-1} \frac{w_j}{w_{i*}} a_{ji*} - \sum_{j=1}^{i-1} \frac{w_j}{w_i} a_{ji} \quad \text{for } i=2,\ldots,n$$

$$\beta_1 = l_{11} + \eta L_{i*} = \sum_{h=2}^{n} a_{1h} - \eta \sum_{j=1}^{i*-1} \frac{w_j}{w_{i*}} a_{ji*}$$

$$\beta_i = l_{ii} + \eta L_{i*} - L_i = \sum_{h=1, h\neq i}^{n} a_{ih} + \eta \sum_{j=1}^{i*-1} \frac{w_j}{w_{i*}} a_{ji*} - \sum_{j=1}^{i-1} \frac{w_j}{w_i} a_{ji} \quad \text{for } i=2,\ldots,n.$$

$$x_i(k+1) = \frac{1}{\beta_i}\left(\sum_{j=1}^{i-1} a_{ij} x_j(k+1) + \alpha_i x_i(k) + \sum_{j=i+1}^{n} a_{ij} x_j(k) \right) \quad \text{for } i=1,\ldots,n \text{ and } k\geq 0.$$

Characterization of the Iterative Schemes

The agents have to perform a start up algorithm before applying the consensus protocol: 2 phases

Assignment phase → each agent receives an identification number *i* and the entries w_i and w_j for each neighbor agent

Communication phase → the agents find out the values of α_i and β_i by a communication protocol.

Characterization of the Iterative Schemes

Start-up algorithm

Assignment phase

A1) **Assign** an order among the agents: each agent is associated with an identification number $id=i$ with $i\in\{1,\ldots,n\}$.

A2) **Assign** to each agent $i\in\{1,\ldots,n\}$ the values w_i and w_j $\forall j\in US_i$.

Communication phase: determining $L_i^{\max}$, α_i *and* β_i.

C1) **Determine** the pair $(i,\ L_i)$: If $i>1$ then **set** $L_i = \sum_{j=1}^{i-1} \frac{w_j}{w_i} a_{ji}$ else **set** $L_1=0$.

C2) **Set** $L_i^{\max} = L_i$

C3) **For** $k=1,n$

C4) **Send** $L_i^{\max}$ to each $j\in N_i$

C5) **Receive** $L_j^{\max}$ from each $j\in N_i$

C6) **For** each $j\in N_i$

If $L_j^{\max} > L_i^{\max}$ **then set** $L_i^{\max} = L_j^{\max}$

End for

C7) **End for**

C8) **Set** $L_{i*} = L_i^{\max}$

C9) **Determine** α_i and β_i according to (23)-(26).

C10) **End**

Characterization of the Iterative Schemes

- If the graph is balanced then w_i=1 for i=1,…,n then the assignment phase consists just in the communication of the updating agent order.
- The agents can autonomously determine the values of α_i and β_i by skipping the communication phase of the startup algorithm.

Outline

- The Consensus Problem
- The Consensus Algorithms
- A New Class of Consensus Algorithms
- Convergence Properties of the Iterative Schemes
- Convergence Performance Analysis
- Conclusions and Future Research

Convergence Performance Analysis

- We consider a network of 20 agents with different topologies.
- The asymptotic convergence properties and the convergence times are evaluated on 1000 randomly generated adjacency matrices.
- For each system, the convergence time *k** is the number of broadcasts such that:

$$\frac{\left\| x(k^*+1) - x(k^*) \right\|_2}{\left\| x(k^*+1) \right\|_2} < 0.01$$

Convergence Performance Analysis

the smaller the iteration matrix eigenvalue λ_2 is, the faster the algorithm is

TABLE II

CONVERGENCE PROPERTIES OF THE CONSENSUS ALGORITHMS FOR UNDIRECTED GRAPHS

Consensus algorithm	$\bar{k}$ *	σ^2	$\bar{\lambda}_2$
P_ε ε=0.5/Δ	18.97	10.60	0.83
Γ_1	6.83	0.54	0.44
Γ	11.26	3.54	0.67

Convergence Performance Analysis

the smaller the iteration matrix eigenvalue λ_2 is, the faster the algorithm is

TABLE III

CONVERGENCE PROPERTIES OF THE CONSENSUS ALGORITHMS FOR DIRECTED GRAPHS

Consensus algorithm	$\bar{k}*$	σ^2	$\bar{\lambda}_2$
P_ε ε=0.5/Δ	17.58	8.28	0.79
Γ_1	5.77	0.22	0.26
Γ	10.59	2.60	0.61

Convergence Performance Analysis

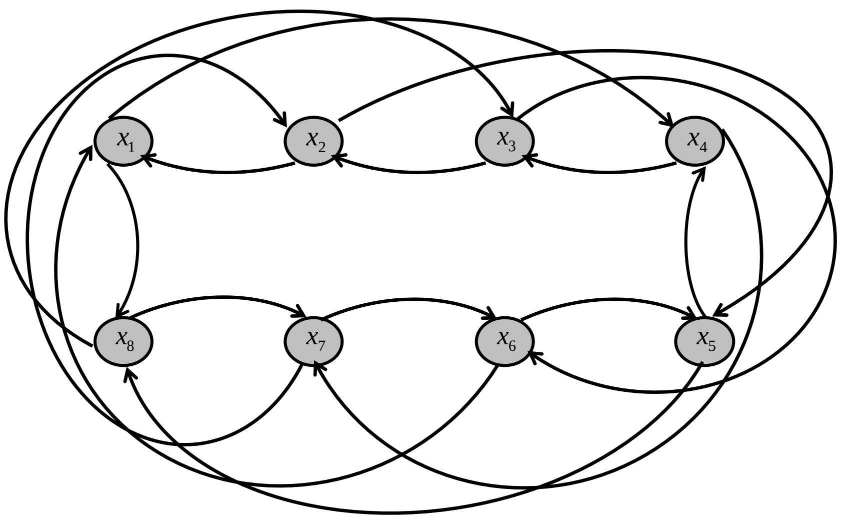
TABLE IV

CONVERGENCE PROPERTIES OF THE CONSENSUS ALGORITHMS FOR PERIODIC GRAPHS

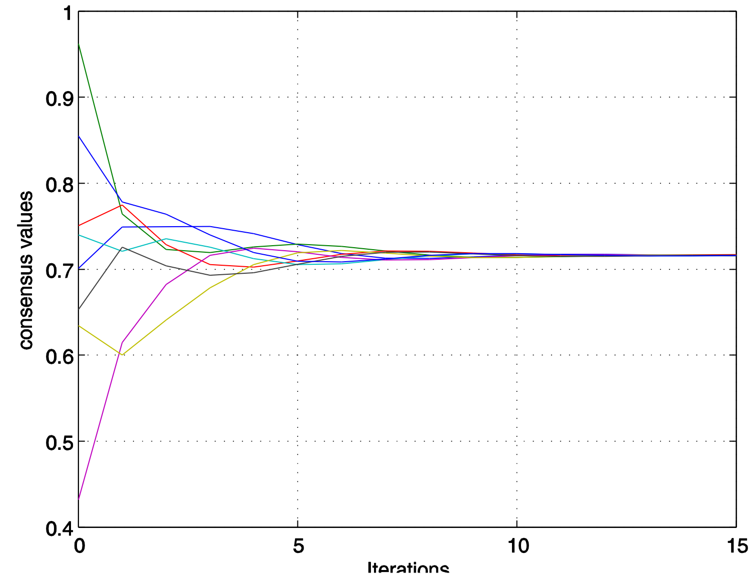
Consensus algorithm		d=2	d=3	d=4	d=6	d=12
P_ε ε=0.5/Δ	k^*	28.1	20.5	18.8	35.3	117.8
	λ_2	0.93	0.85	0.73	0.87	0.97
Γ_1	k^*	25.8	18.2	19.6	35.4	118.4
	λ_2	0.91	0.80	0.75	0.87	0.97
Γ	k^*	21.2	13.8	10.5	10.3	103.4
	λ_2	0.87	0.72	0.57	0.57	0.96

Convergence Performance Analysis

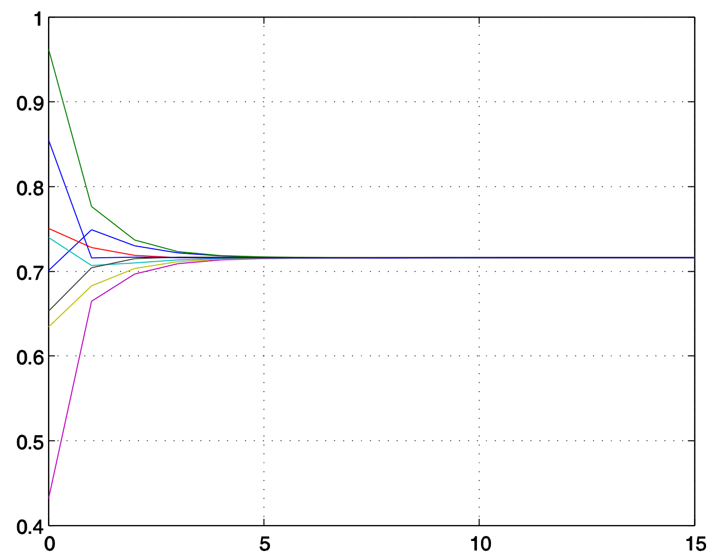
G_4



Classical protocol



New protocol

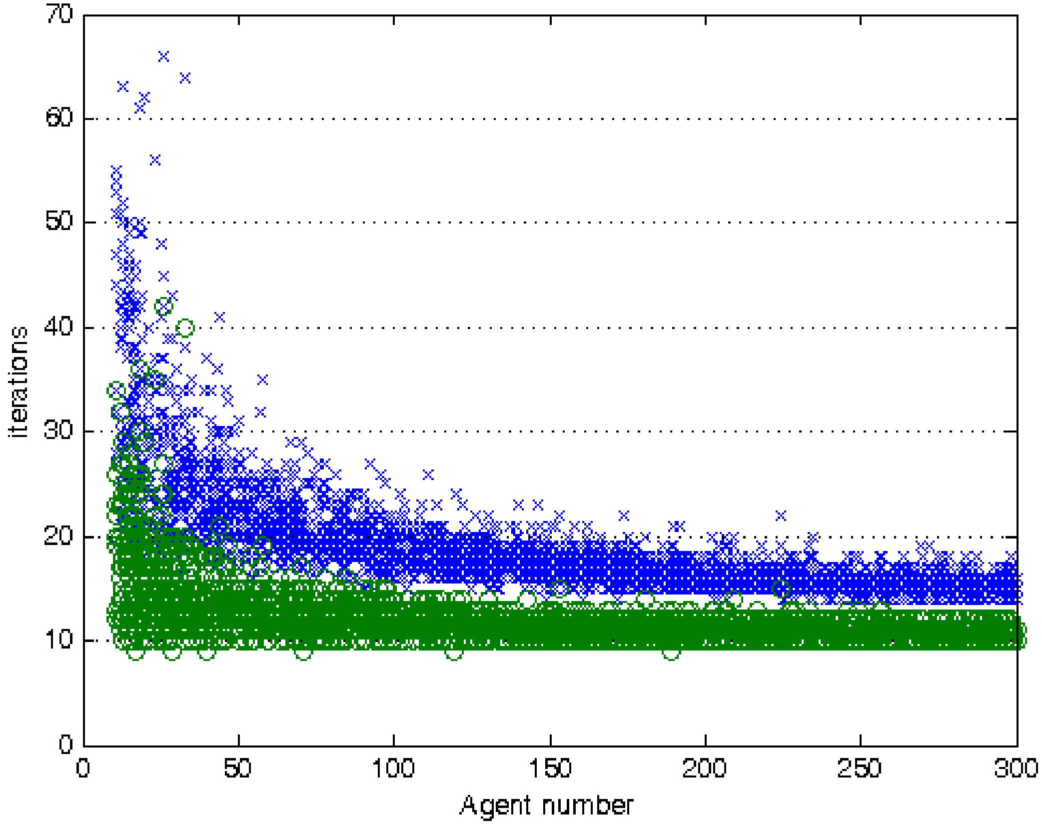


Convergence Performance Analysis

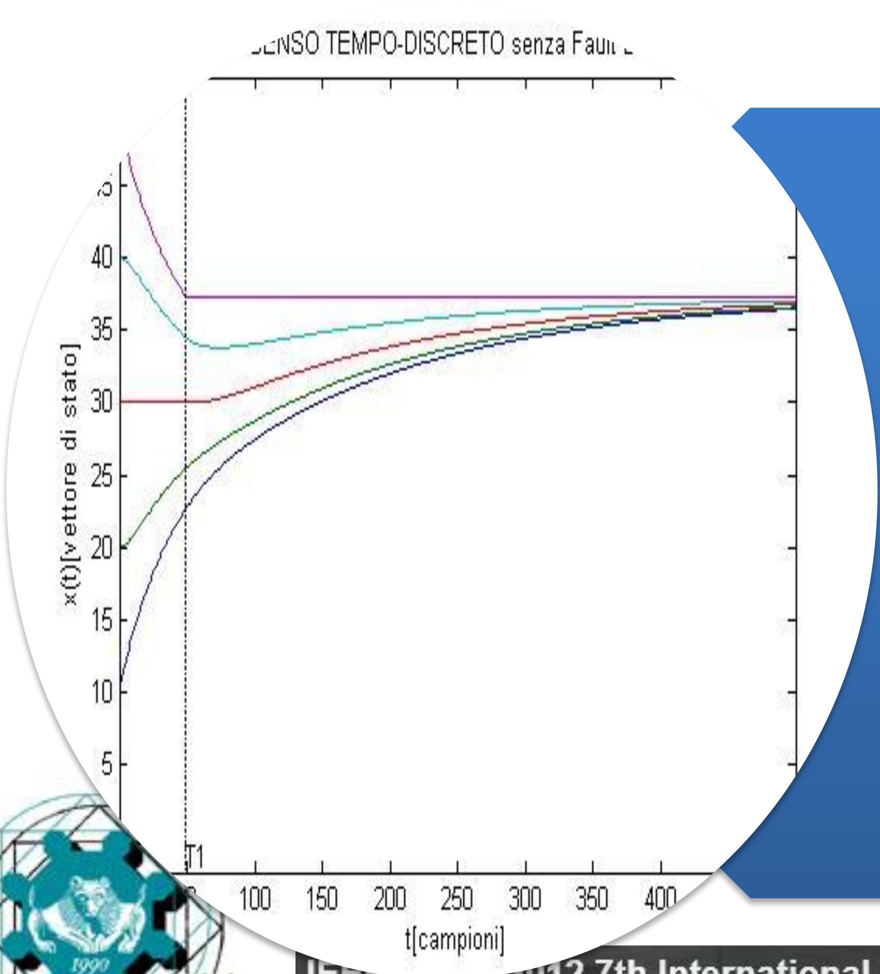
- Simulation results obtained by applying the classic algorithm with $\varepsilon=0.5/\Delta$ (‘ × ’) and the proposed algorithm (‘○’).
- We generate the random strongly connected graph $G(V,E)$ with $n\in[10, 300]$ nodes generated with uniform probability.
- Each node $i\in V$ communicates with node $j\in V$ with probability 0.3.
- The initial state $x(0)$ is selected by choosing each component independently as a uniform random variable over [0,1].

Convergence Performance Analysis

- Classic algorithm: ‘ × ’, The proposed algorithm ‘○’.
- The outcomes for 5000 values of the number of nodes *n*
  - the convergence time of the proposed algorithm is lower than the one of the standard algorithm.



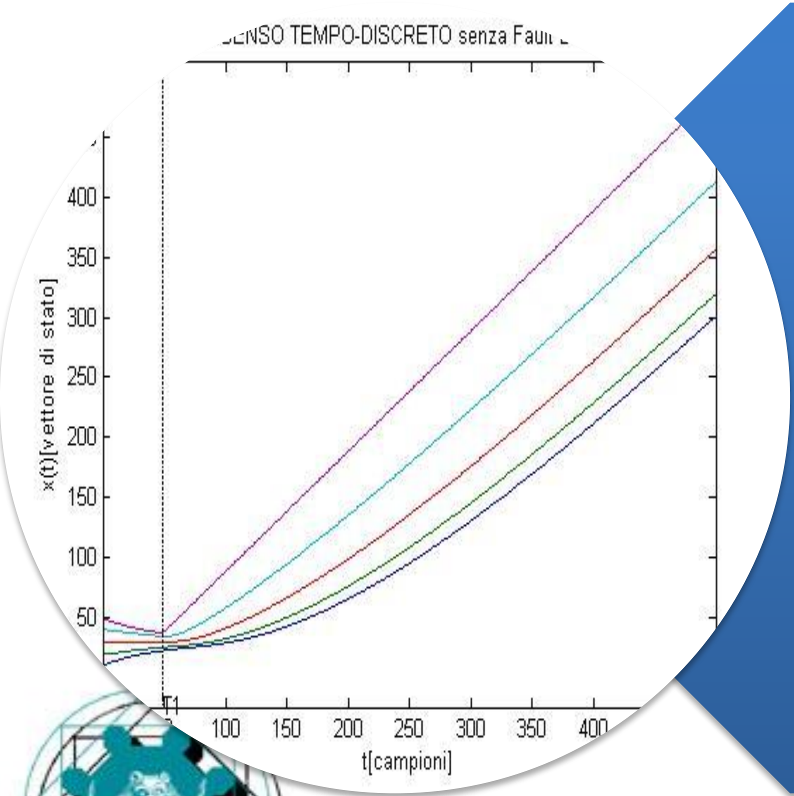
Other problems: decentralized diagnosis of faults



Stuck at: : This fault is characterized by a node that doesn't update anymore its state but remains visible to its neighbors.

The healty nodes tend to follow the state values of the faulty node.

Other problems: decentralized diagnosis of faults



Divergence fault: this fault is characterized by an indefinite constant increment (or decrement) of the node's state.

This kind of fault can be due for instance to software or hardware bugs and it prevents the network to converge toward a common value.

Other problems: decentralized diagnosis of faults

The diagnosis and recovery approach can be solved by suitable algorithms composed of 3 phases

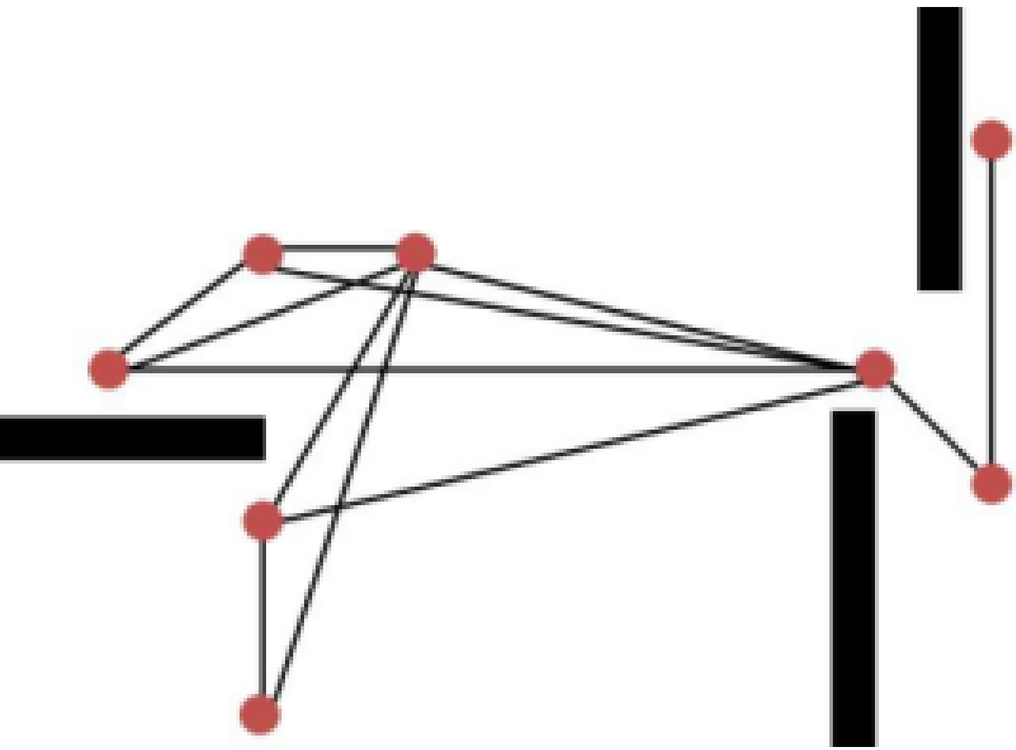
FAULT DETECTION

FAULT IDENTIFICATION

RECOVERY

The network converges to the average states of the healty nodes if the graph of the healty nodes is strong connected

Other problems: dynamic network topologies



Networked systems can possess a dynamic topology that is time-variant due to node and link failures/creations, packet-loss, state-dependence, reconfiguration, evolution.

Networked systems with a dynamic topology are commonly known as switching networks that can be modeled using dynamic graphs:

G(t)=(V,E(t))

The edge set E(t) and the adjacency matrix A(t) are time-variant.

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Conclusions and future research

- Consensus algorithms for multi-agent networked systems and sensor swarms are an exciting frontier of research.
- Some theoretical frameworks are provided for consensus algorithms for networked multi-agent systems with fixed or dynamic topology and directed information flow.
- Consensus problems include synchronization of oscillators, flocking, cooperation can be solved by Markov processes, gossip-based algorithms, load balancing in networks, distributed optimization strategies.

Conclusions and future research

- We investigated **new and fast alignment protocols** to be applied to the discrete time model of consensus networks.
- We propose a class of consensus algorithms that are based on a **triangular splitting** of the standard iteration matrix.
- The **convergence** of the proposed algorithms is proved in the framework of **non-negative matrix theory**.
- A set of tests shows that the presented algorithms exhibit **good performances** even in the cases in which the standard consensus protocols converge slowly.

Conclusions and future research

Some open issues for future research:

- New efficient techniques to solve **decentralized fault detection, diagnosis and recovery of consensus**
- New consensus protocols for **complex distributed task assignment problems**
- **New gossip algorithms** for quantized consensus

New Consensus Protocols for Agent Networks with Discrete Time Dynamics and Distributed Task Assignment

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