Urszula Kanturska

Making decisions in hazardous transport networks

Michael G H Bell

Imperial College London

Introduction

Imperial College London

- Aim
  - Reliable transport in uncertain networks
- Approach
  - Game theory: Demon(s) try to disrupt trips
    - Single demon: Low probability – High consequence (LPHC)
    - Multiple demons: High probability – Low Consequence (HPLC)
- Questions
  - Where will demon(s) strike? Critical links
  - How to reduce the risk? Strategy
- Solution
  - LPHC: Olympic Route Network
  - HPLC: Vehicle navigation

ARUP

Presentation Outline

- PART 1 Introduction to the approach
  - Uncertainty and risk
  - Game theory
- PART 2 Example: Olympic route network
  - Single demon game
  - Benefits from routing strategy
  - Benefits from defence strategy
- PART 3 Example: Vehicle navigation
  - Multiple demon game
  - Hyperstar algorithm
  - Time-dependent vehicle navigation

Transport risk factors

Imperial College London

Uncertainty about incident probability…

…so focus on consequence minimisation!

Risk = incident probability x incident impact

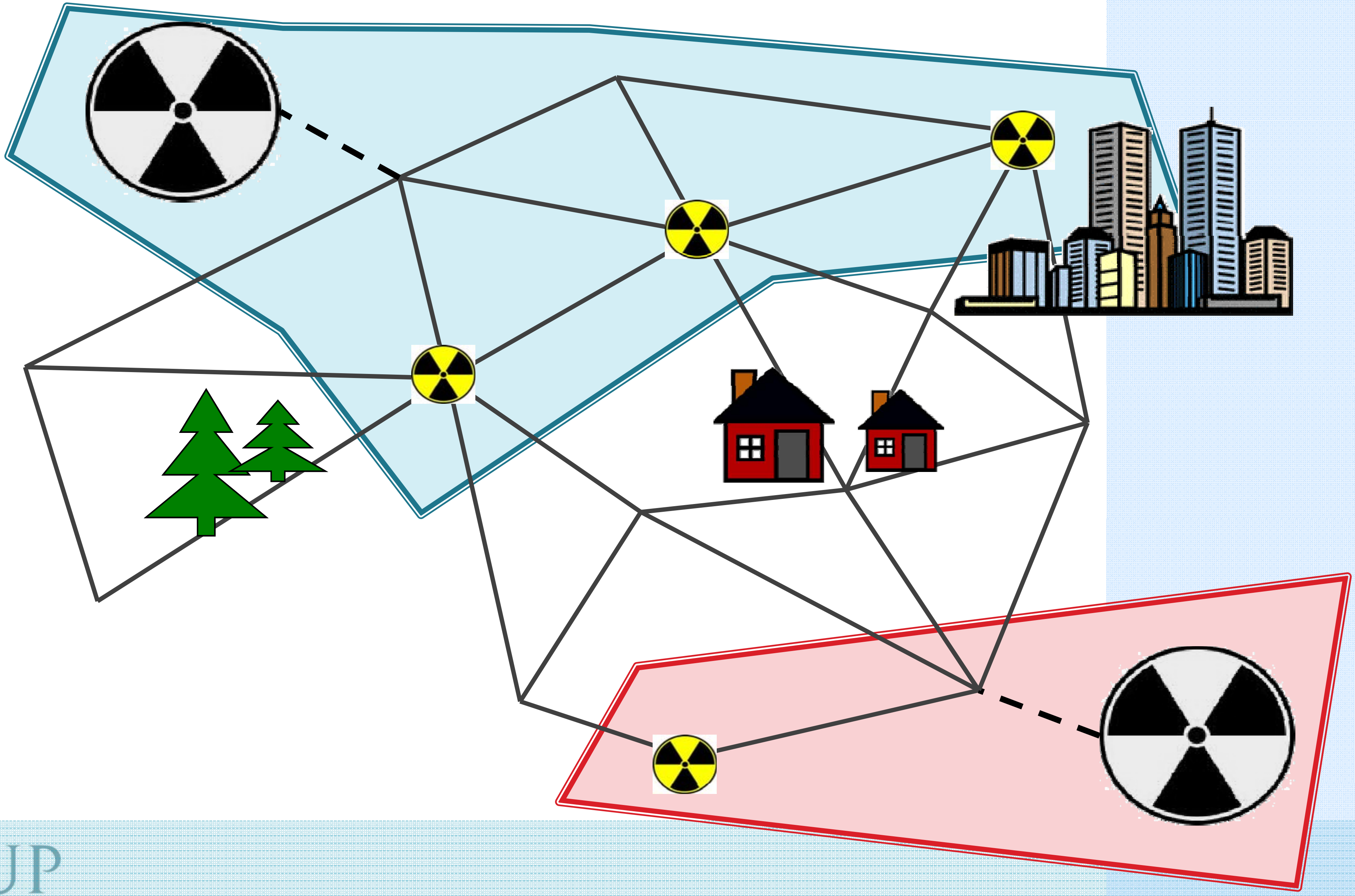
low probability | **high impact**

low probability	high impact
Community protest	Residents
Terrorist attack	Other travellers
Road accident	Environment
Weather	Package quality
Traffic levels	Amount of waste
Time of the day	Frequency of dispatches
Route	Time of the day
	Route
	Location – Allocation

scope for further risk reduction

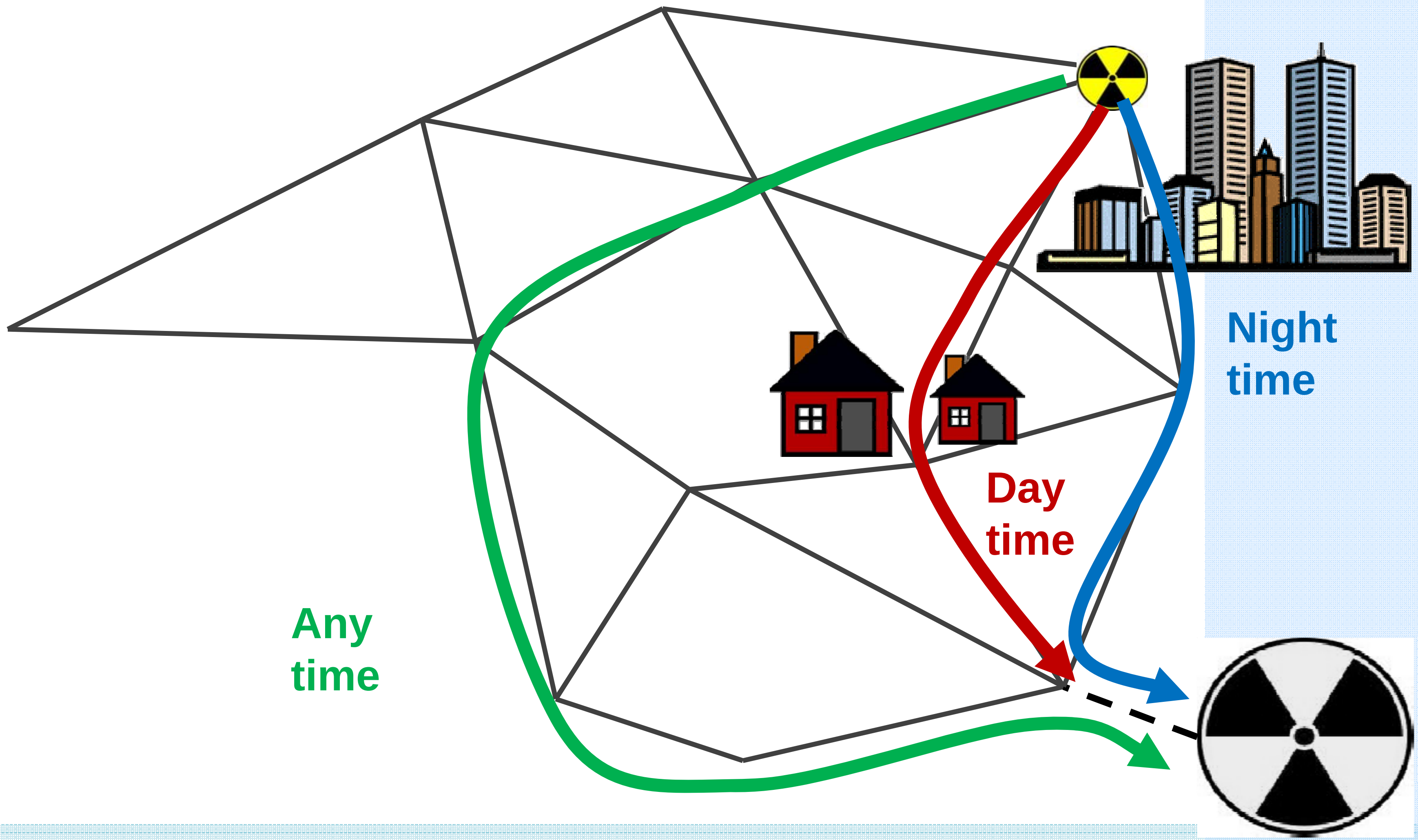
ARUP

Example: disposal site and source allocation



Example: Combined routing and scheduling

Imperial College London



ARUP

PART 1
Research background

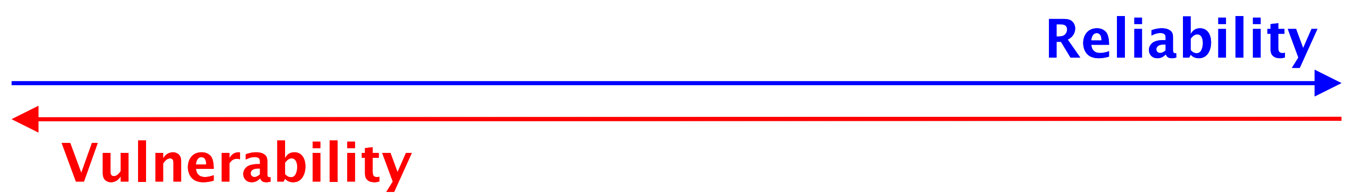
Uncertainty and Game Theory

Reliability – Vulnerability – Risk

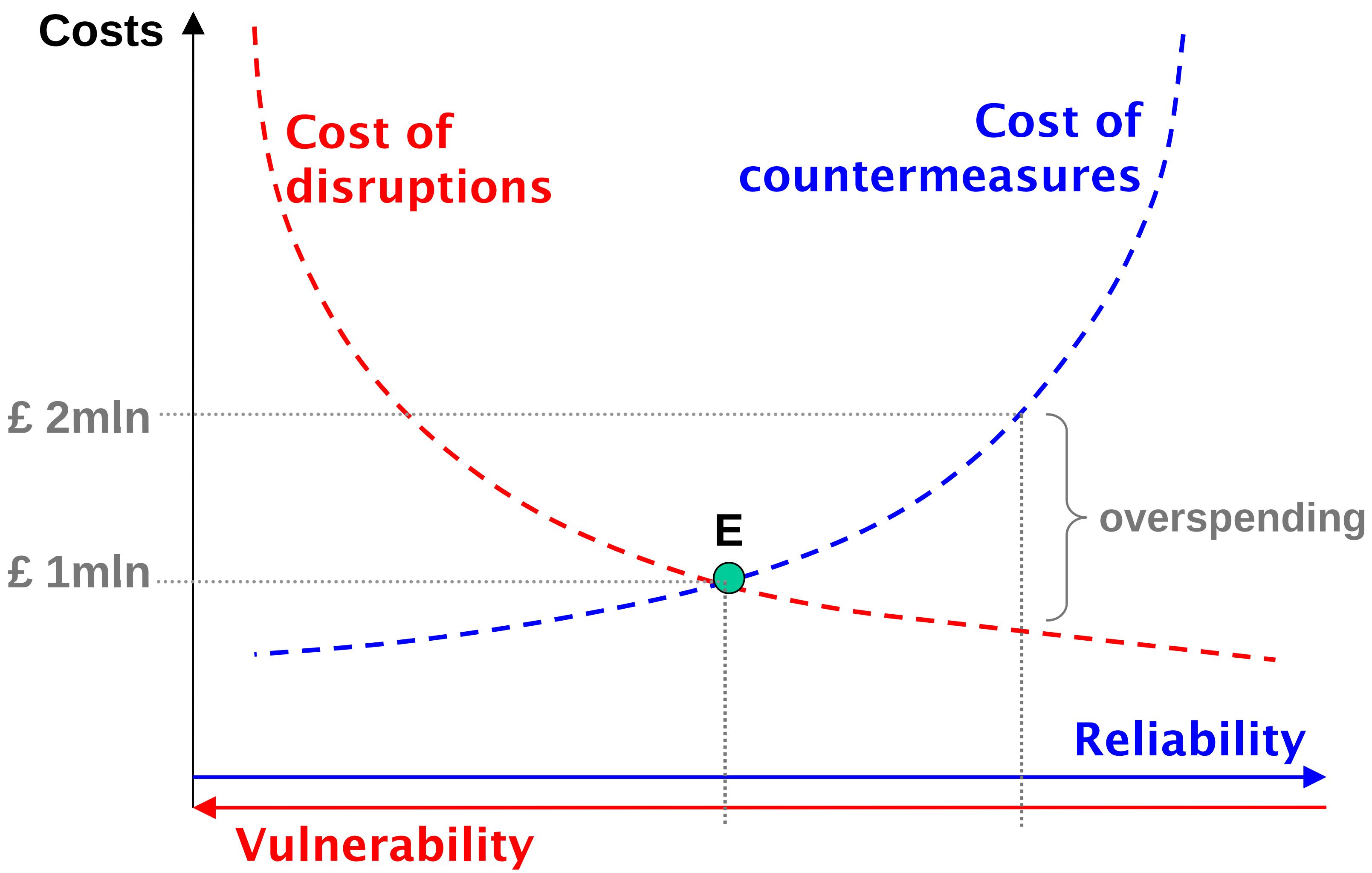
- Security = acceptable level of risk
- Risk = potential loss
- Risk = hazard/threat x vulnerability

EXTERNAL INTERNAL

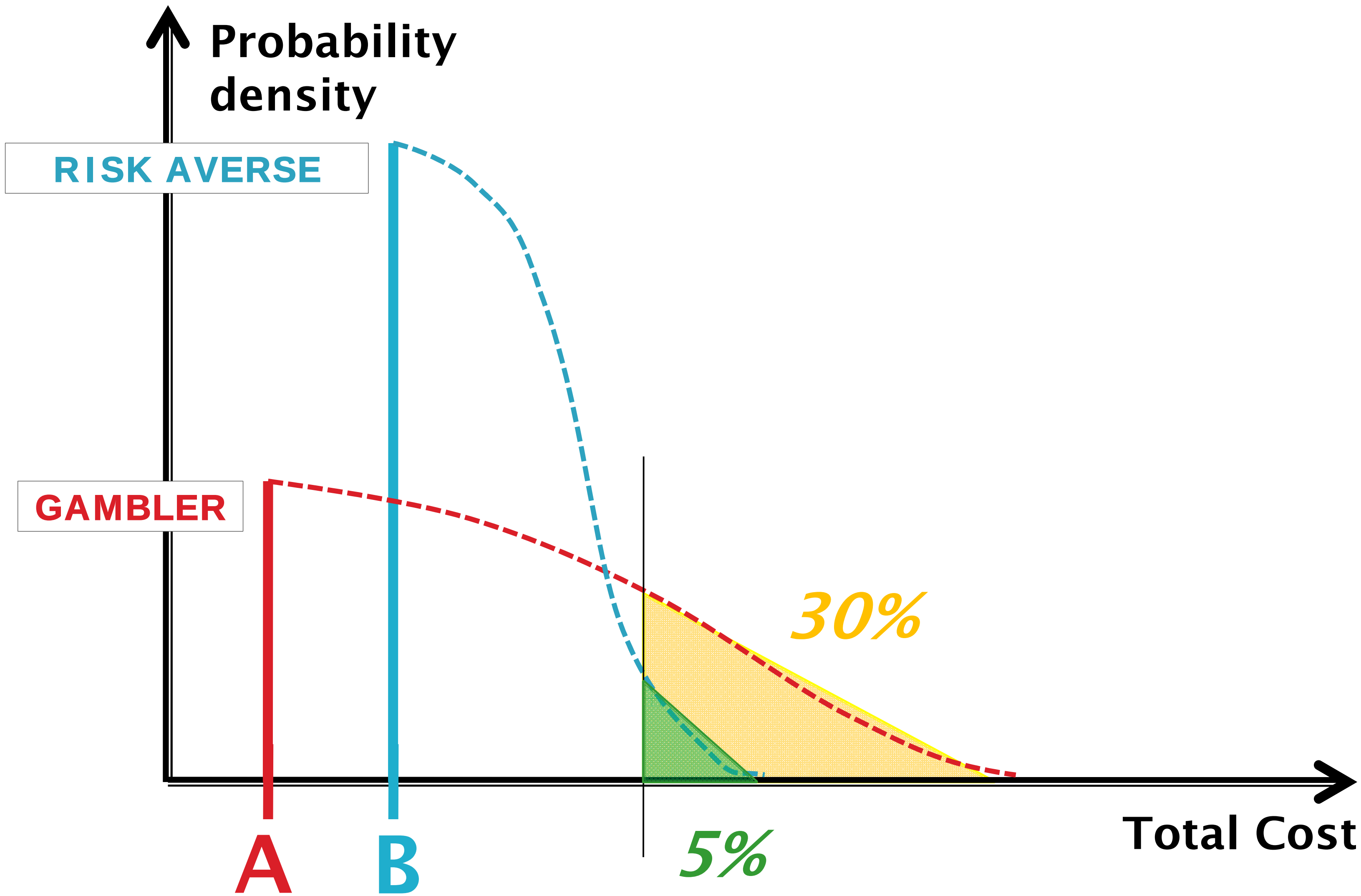
- Vulnerability =inability to avoid potential harm
- Reliability = stability in the quality of service



Reliability vs Vulnerability



Decisions under uncertainty



Investment +
Consequence

Risk averseness and game theory

Demon

- What to attack?

Dispatcher

- Which route to take?

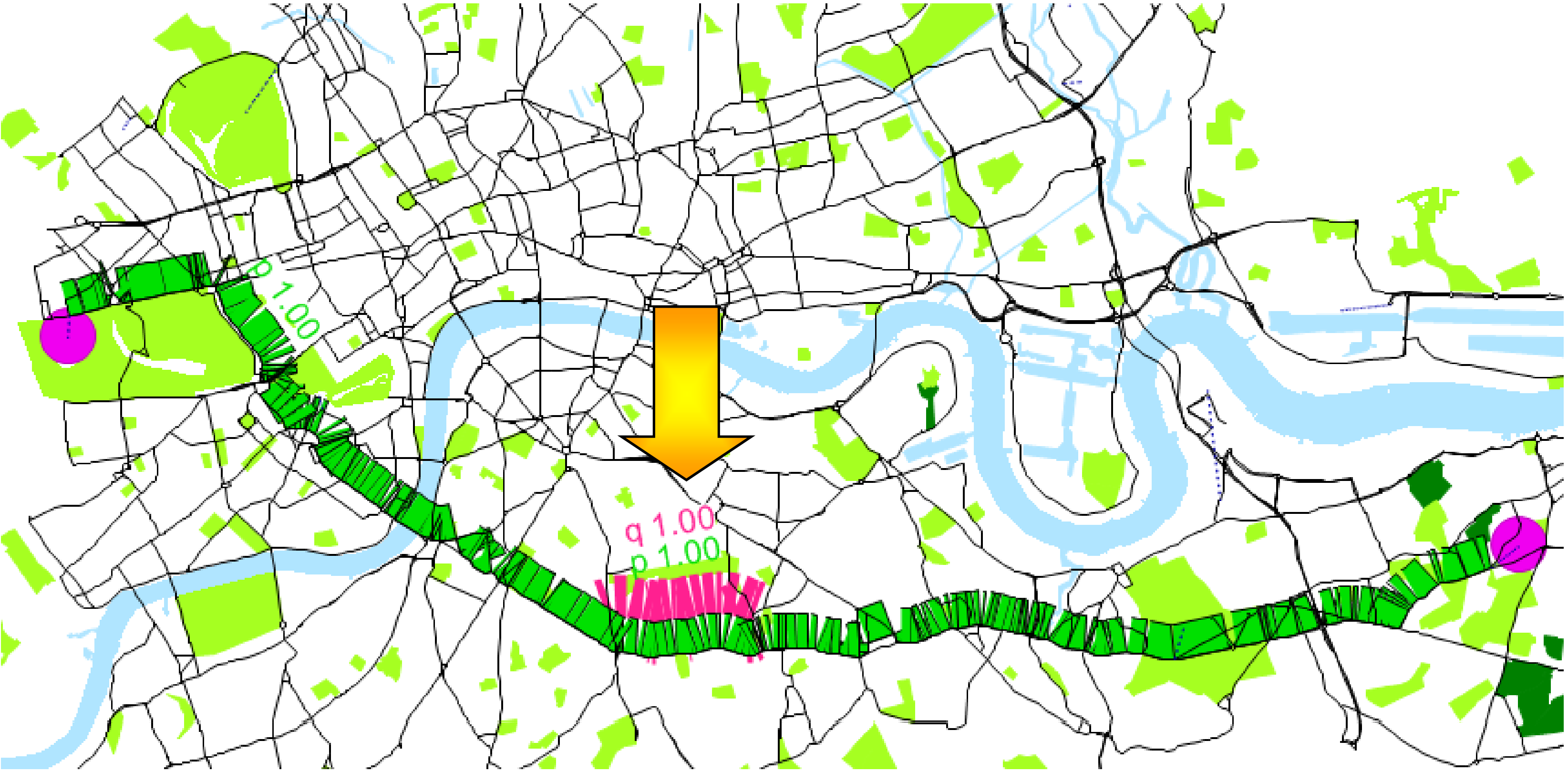
GAMBLER

RISK AVERSE

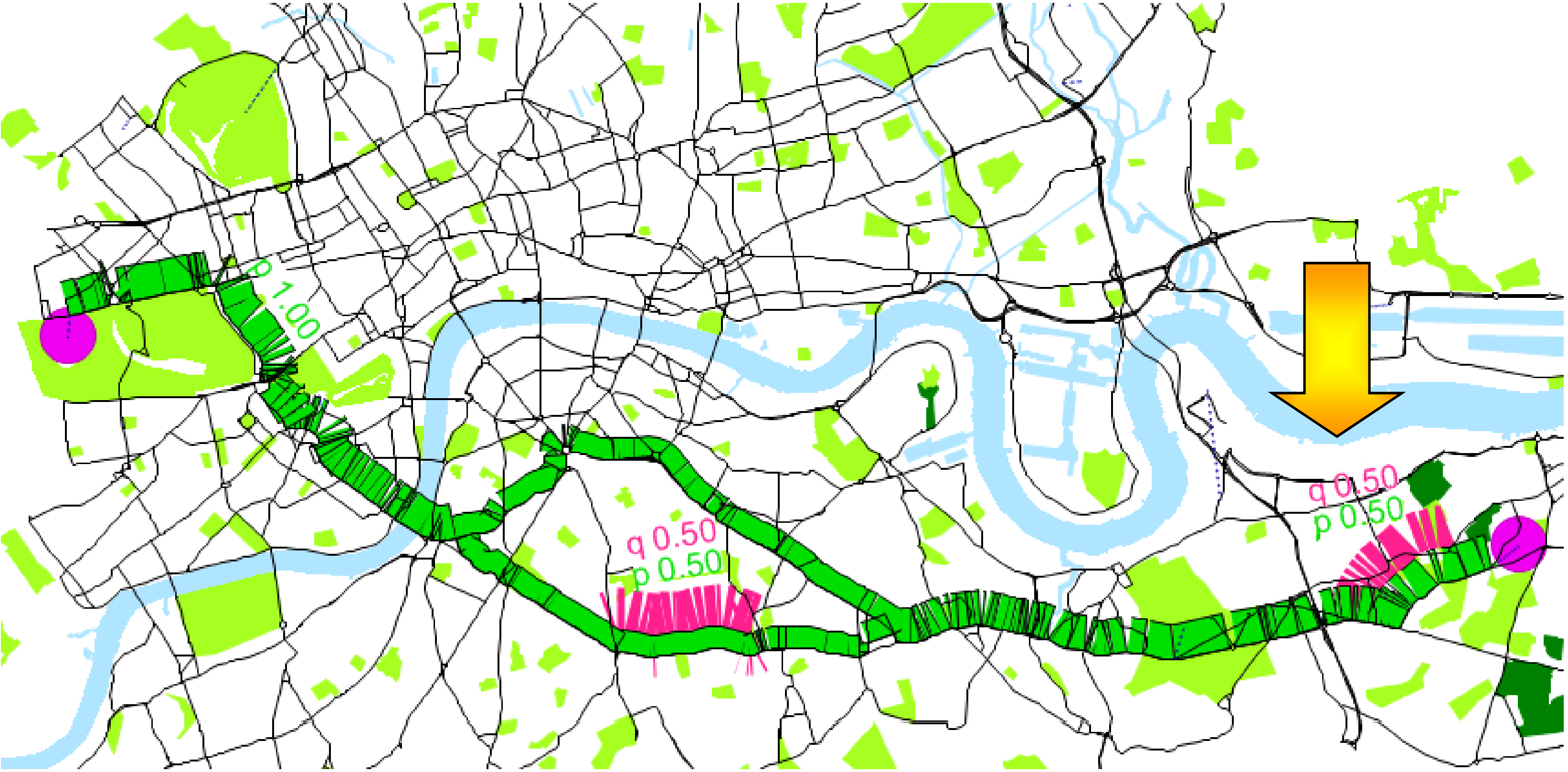
£	S1	S2	S3	S4	MAX	MIN
T1	12	−1	1	0	12	−1
T2	5	1	7	−20	7	−20
T3	3	2	4	3	4	2
T4	−16	0	0	6	6	−16
MIN	−16	−1	0	−20		
MAX	12	2	7	6		

How the game works? – Round1

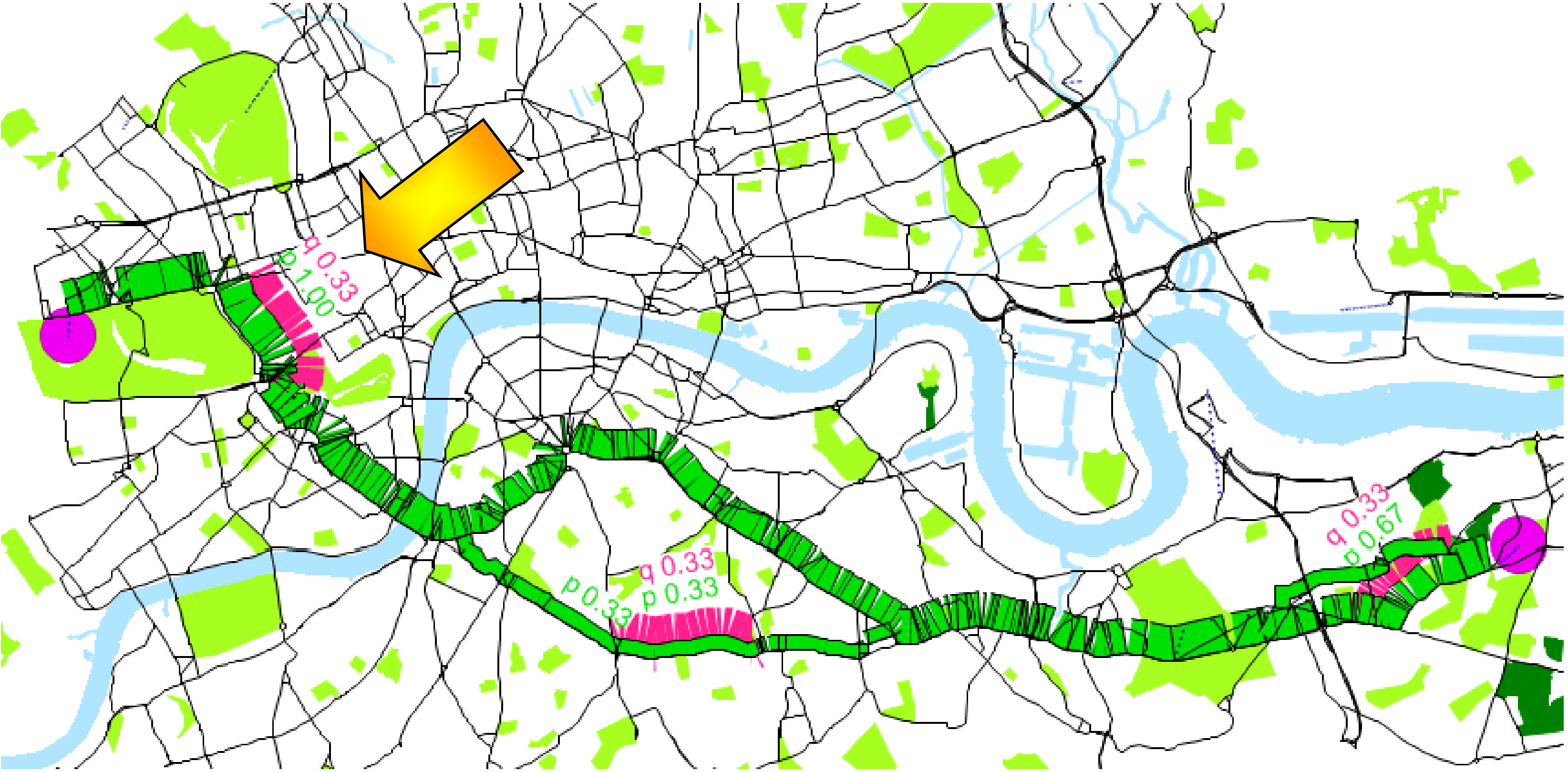
Disruption
= increase
in cost



How the game works? – Round 2



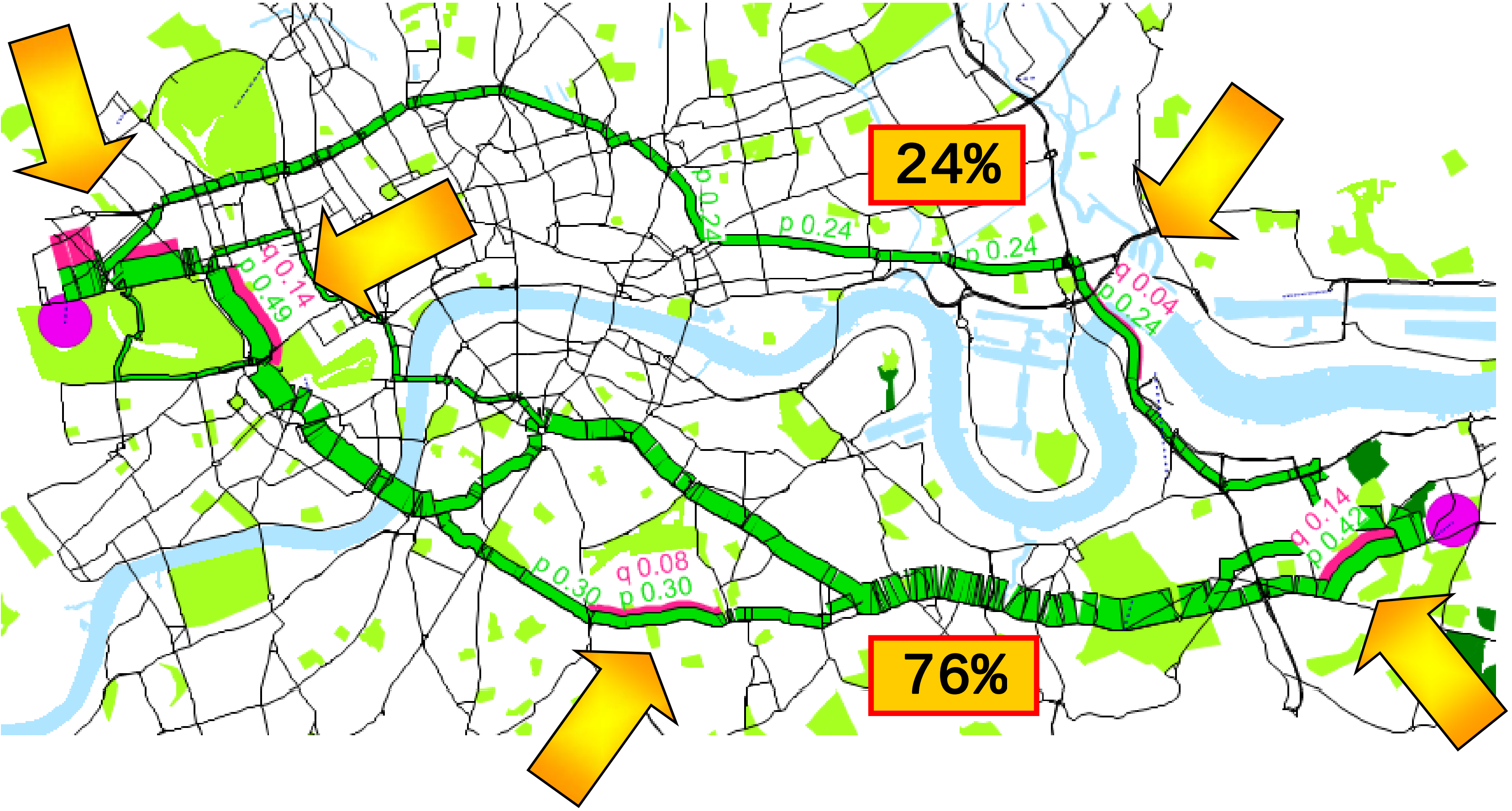
How the game works? – Round 3



How the game works? – Equilibrium

Imperial College London

Bar width = probability value



At the solution

- Routes used
  - Only routes attractive to the dispatcher are generated
  - Routes with minimum expected cost
  - Link use probabilities

    → Safest path choice frequency

- Links attacked
  - Only links attractive to the demon are attacked
  - Links with maximum expected loss
  - Only links with non-zero link use probability
  - Link failure probabilities

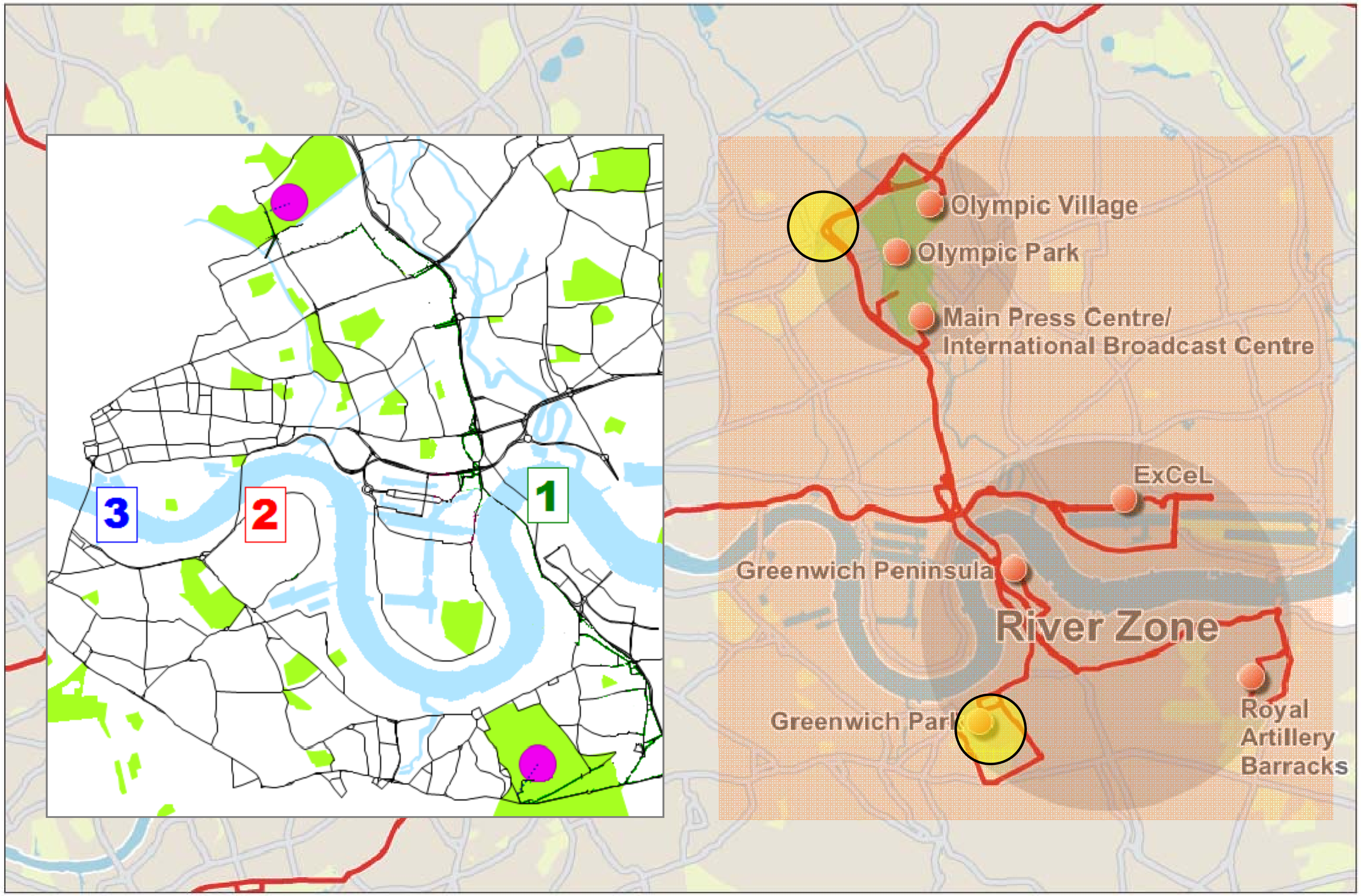
    → Critical links

Imperial College London

PART 2
Application to Olympic routes

» Routing & Defence Strategies

Transport game applied to ORN



Analysis of the ORN network

- Single routing
  - Without disruption
  - With disruption
    - minor k=2
    - major k=1,000,000

- Multiple routing
  - Without disruption
  - With disruption
    - minor k=2
    - major k=1,000,000

- Multiple routing with active defence
  - With disruption
    - major k=1,000,000

Potential losses

Potential benefits

Shortest path

- Cost 727 sec

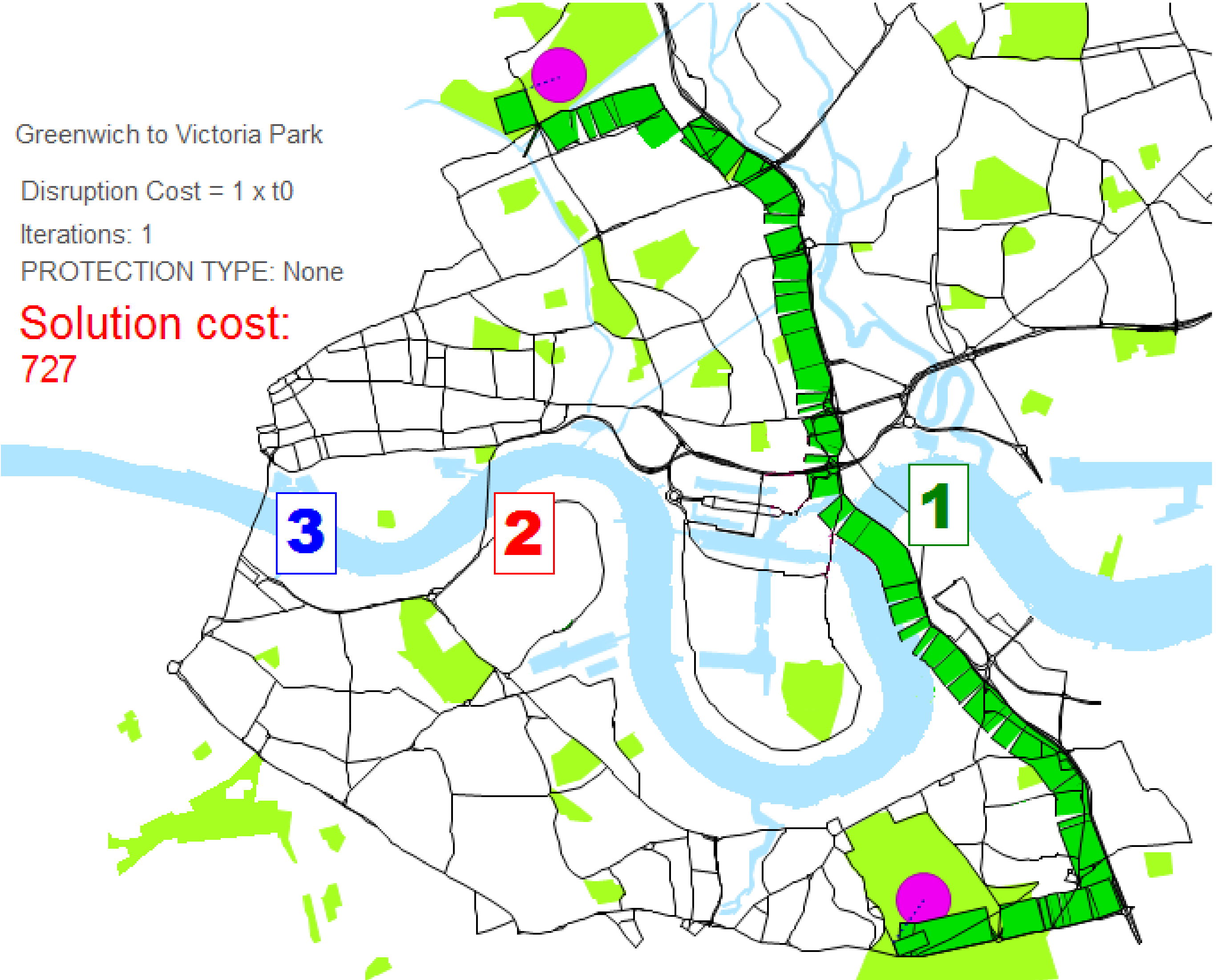
Greenwich to Victoria Park

Disruption Cost = 1 x t0

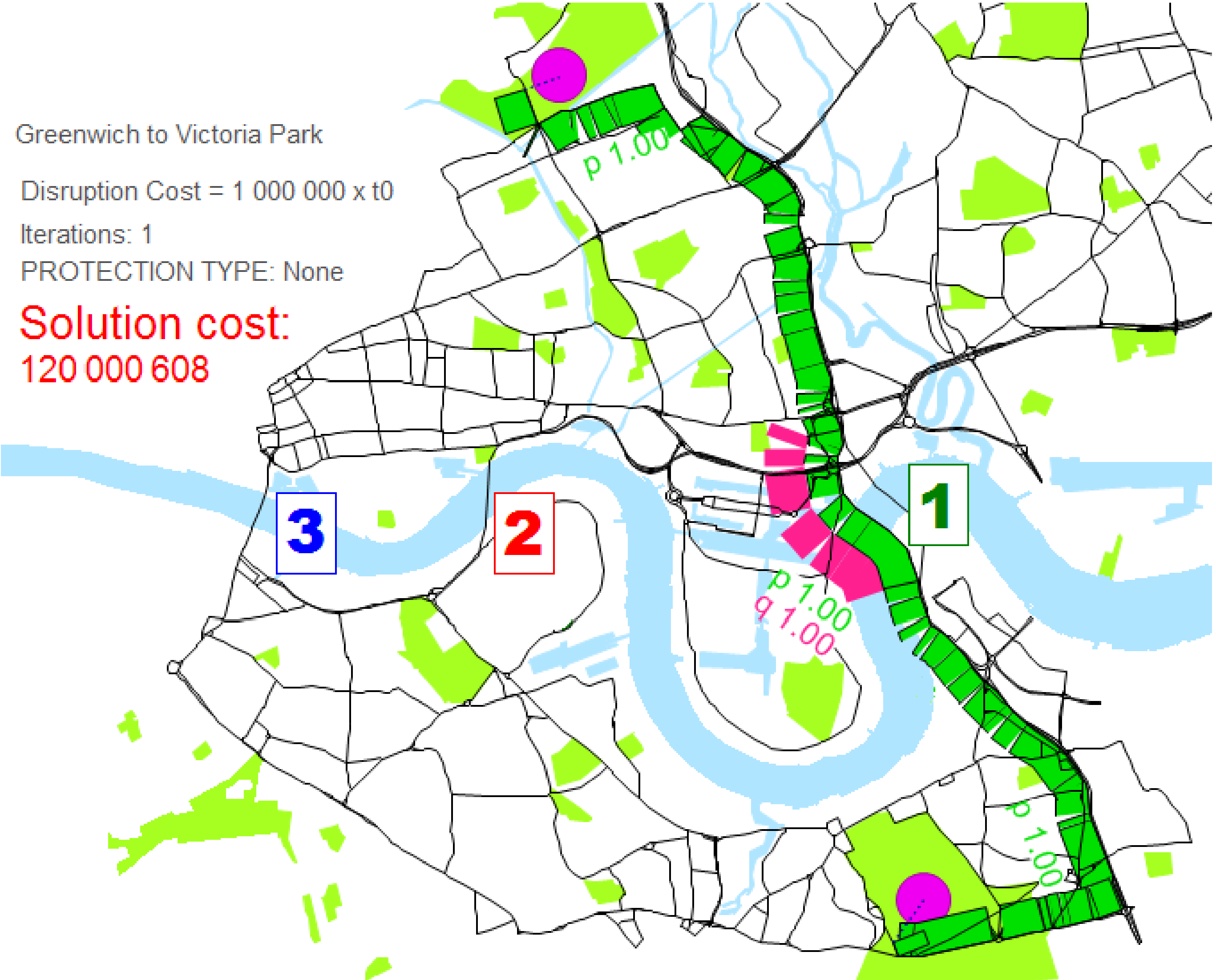
Iterations: 1

PROTECTION TYPE: None

Solution cost:
727



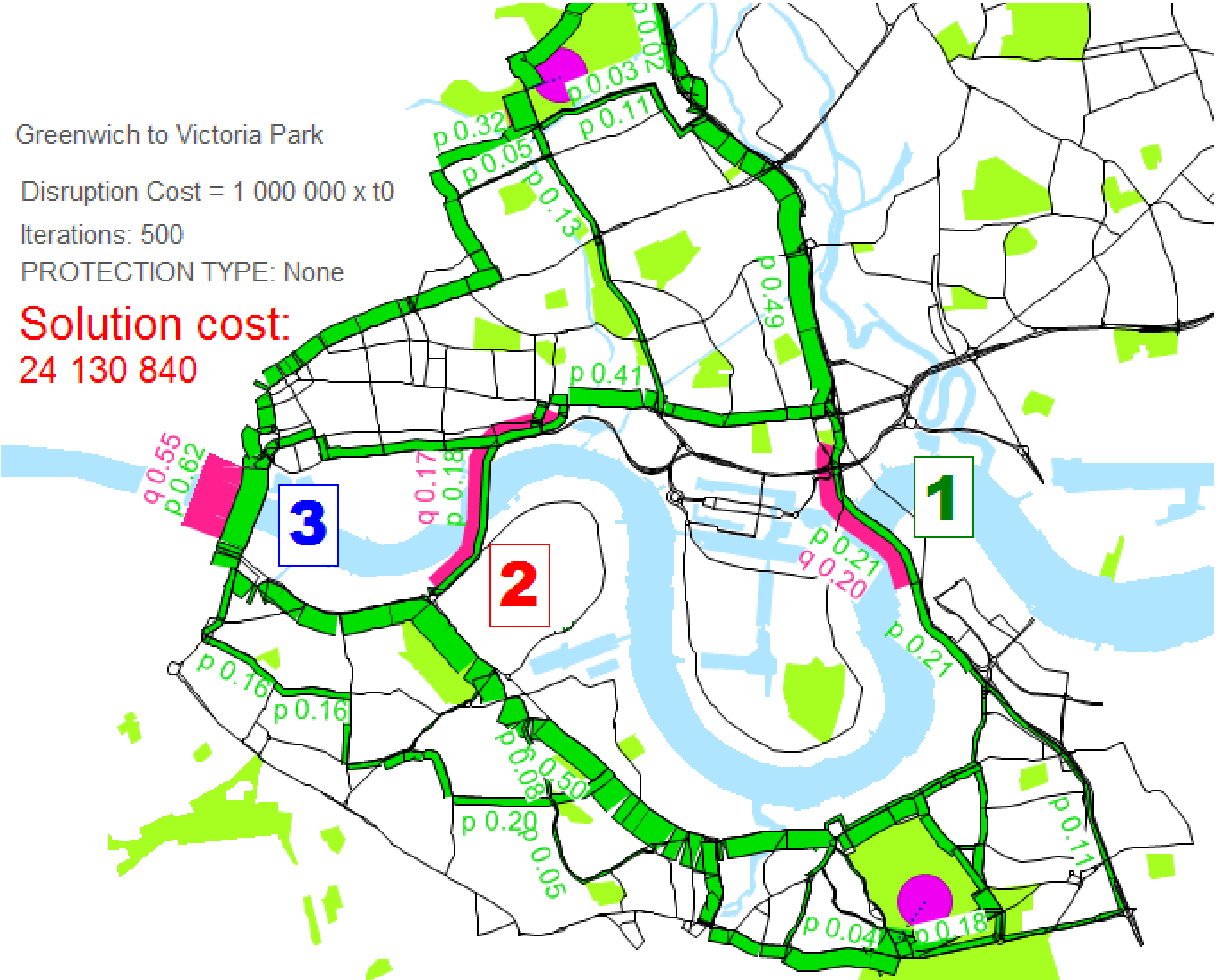
Single routing + major disruption



- Cost 727 sec
- Cost 120m sec

Multiple routing+major disruption

Imperial College London



- Cost 727 sec
- Cost 120m sec
- Cost 24m sec

Saving
80%

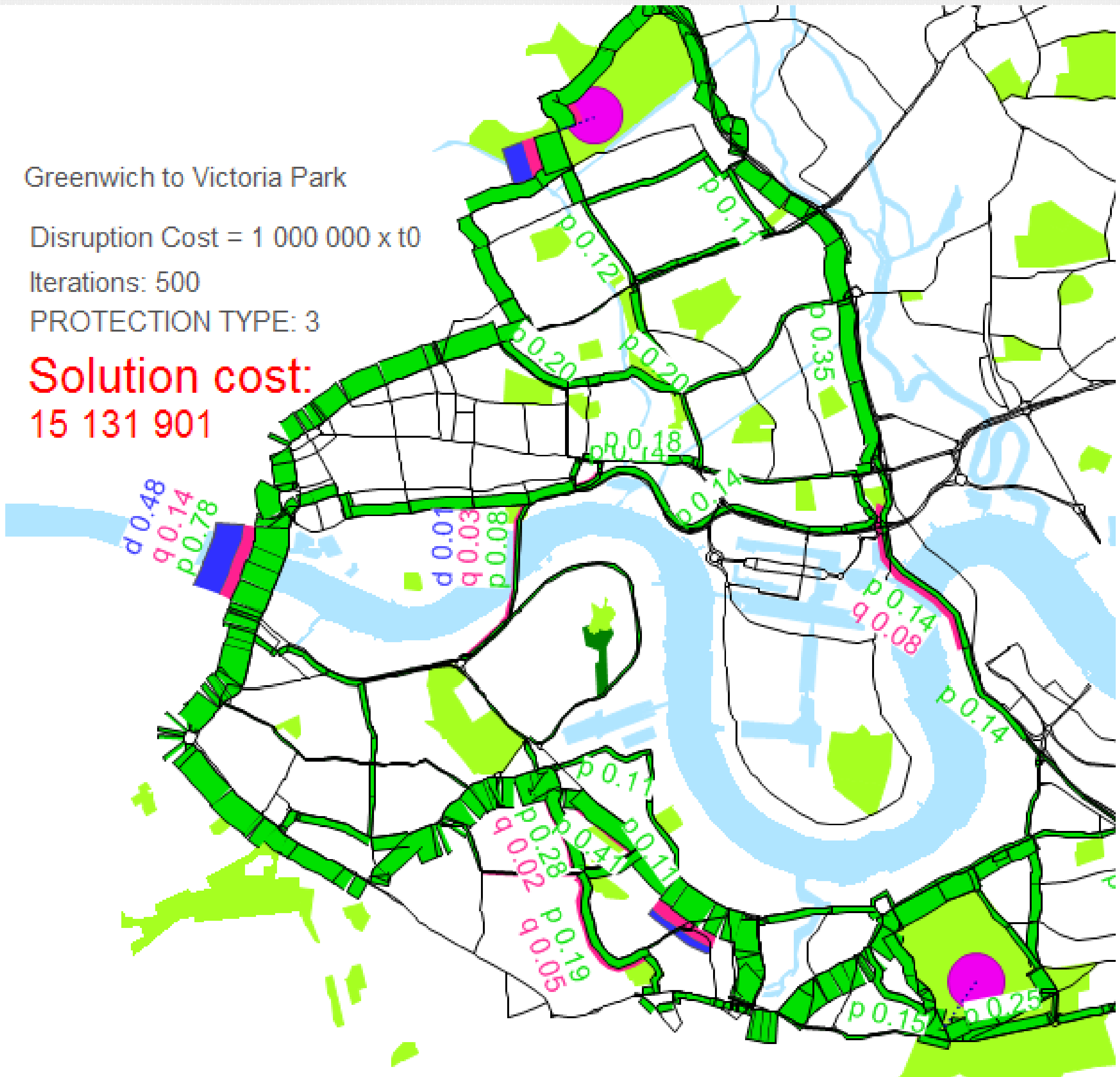
Results summary 1

Major Disruption		
Total Cost (sec)	Does not Happen	Does Happen
A Single route	727	120 m
B Optimal routes	1102	24 m

Comments 1

- Significant benefits from multiple routing at a relative low cost
- Multiple routing mitigates the risk of a serious disruption
- Routes with least expected costs are generated
- Number of routes depends on the size of potential losses

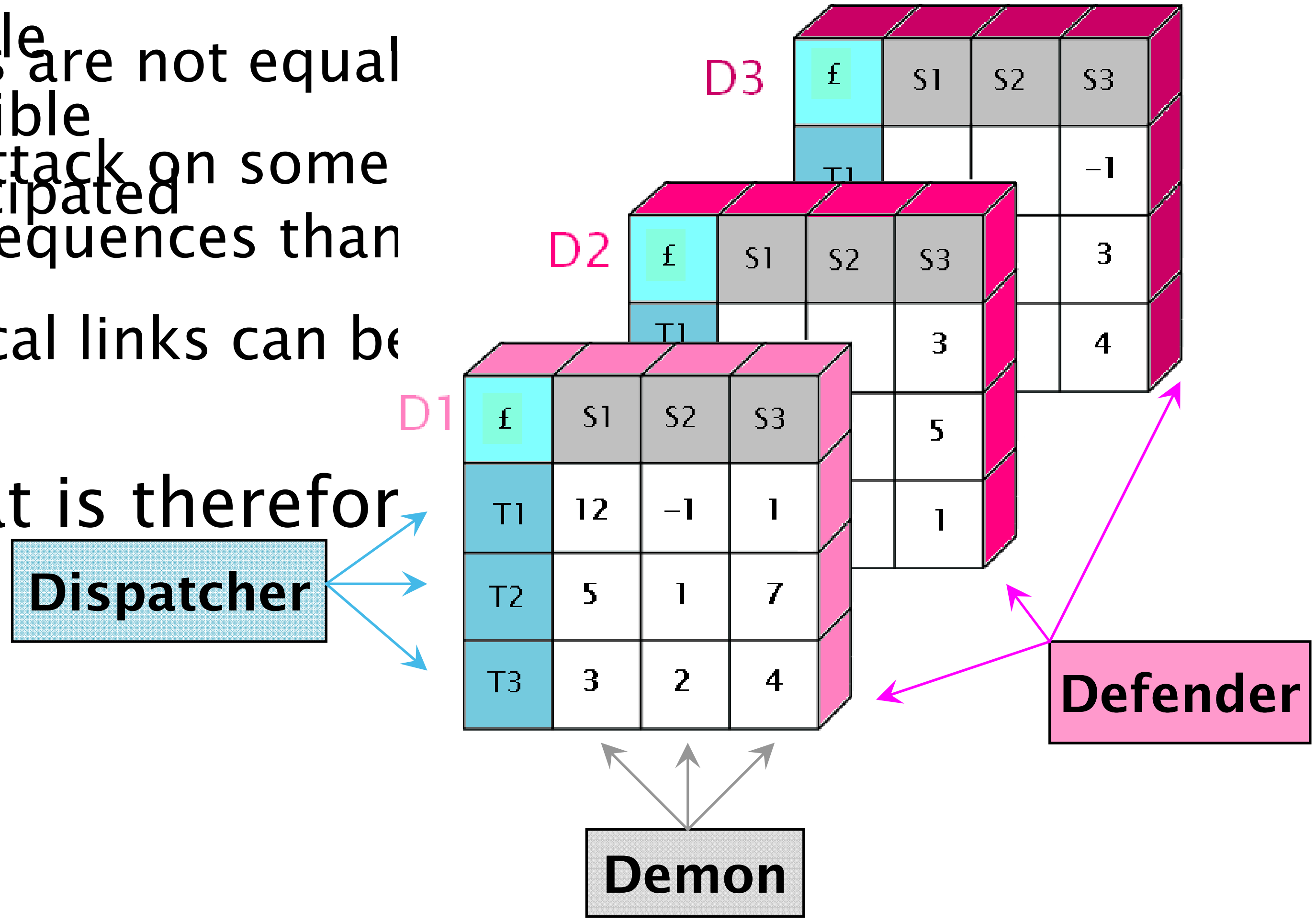
Anticipated defence



- Cost 24m sec
- Cost 15m sec

Transport games with defence

- Considered defence types
  - Visible
  - Invisible
  - Anticipated
- Presented solutions show that:
  - Links are not equal
  - An attack on some consequences than
  - Critical links can be
- ...what is therefor



Anticipated defence – path choice

Imperial College London

	Blackwall Tunnel		Rotherhithe Tunnel		Tower Bridge	
DEFENCE	NO	YES	NO	YES	NO	YES
Link Use	21%	14%	18%	8%	62%	78%
Link Attack	20%	8%	17%	3%	55%	14%
Link Defence	–	0%	–	1%	–	48%

Results summary 2

Cost [million sec]	Defence type			
	Routing only	Visible	Invisible	Anticipated
Solution Cost	24	17	10	15
Benefit	–	7	14	9
% of the SC	–	30%	58%	37%

Comments 2

- Defence influences the optimal routing
- Invisible defence yields max benefits
- It is most beneficial to protect river crossings, in particular Tower Bridge.
- Even if only one link is protected, the expected cost can be significantly reduced

Application of the method

Imperial College London

Strategic	Operational	Navigation
Find critical links	Check what happens if some links are no longer available	Produce individual routing plans for drivers
Estimate costs of various scenarios	Produce contingency routes updated according to road conditions	Real time update using on-line traffic information
Establish optimal routing and defence strategies		

Refinements

- Flow dependent link costs
- Joint examination of multiple OD
- Link failure affecting both directions

- Attack and defence of multiple links
- Budget constraints
- Deceptive strategies

- Dynamic effects

Conclusions

- Multiple routing is a rational measure to distribute risk

- Potential for application

- Optimal routing & defence strategies bring significant quantifiable benefits

Imperial College London

PART 3
Application to vehicle navigation

- Strategic & Operational Planning and Navigation

Introduction

- LPHC implies one demon
- HPLC implies multiple demons
- HPLC:
  - Place a demon at every node
  - Solve by a version of the Spiess and Florian hyperpath algorithm
  - Accelerated by node potentials

Assumptions

- Every link $a \in A$ has a cost of use c_a under normal operating conditions
- There is an additional cost of use d_a if the link is congested
- Worst case: On exiting any node $i \in N$, one link is degraded
- Seek link use probabilities that minimise expect travel cost subject to worst case link congestion probabilities

Demon games and the minmax exposure principle

- Every node has a demon with the ability to fail one outgoing link

- Consider a zero sum game, where each demon can select one outgoing link a to impose d_a and the dispatcher seeks a least cost route with respect to c_a and expectation of d_a (Schmoecker et al., 2009)

- Find the mixed strategy Nash equilibrium by:

$$Min_{\mathbf{p}} \left(\sum_{a \in A} c_a p_{as} + Max_{\mathbf{q}} \sum_{a \in A} q_{as} d_a p_{as} \right)$$

Hypertrees and hyperpaths

- Probability q^*_{as} measures link criticality
- Links with probability $p^*_{as} > 0$ define the *hypertree* to *s*
- $p^*_{as} > 0 \Leftrightarrow q^*_{as} > 0$ and $p^*_{as} = 1 \Leftrightarrow q^*_{as} = 1$
- The hyperpath cost is

$$u_{rs} = \sum_{a \in HP(r,s)} c_a p^*_{as} + \sum_{a \in HP(r,s)} q^*_{as} d_a p^*_{as}$$

Hyperpath generation

Imperial College London

$$Min_{\mathbf{p},\mathbf{w}} \sum_{s \in S} \left(\sum_{a \in A} c_a p_{as} + \sum_{i \in I} w_{is} \right)$$

subject to

$$\sum_{a \in A_i^+} p_{as} - \sum_{a \in A_i^-} p_{as} = g_{is}, \forall i \in I, s \in S$$

$$w_{is} \geq p_{as} d_a, \forall a \in A_i^-, i \in I, s \in S$$

$$p_{as} \geq 0, \forall a \in A, r \in R, s \in S$$

ARUP

Dijkstra's algorithm

1. Start at s and set $u_j = \infty$ for $j \neq s$ and $u_s = 0$
2. Put s in OPEN
3. Search OPEN for smallest u_i
4. For nodes j reached from i if $u_j > u_i + c_{ij}$ then $u_j = u_i + c_{ij}$
5. Put nodes j in OPEN and transfer i to CLOSED
6. Return to Step 3 until r in CLOSED

A* algorithm

1. Start at s and set $u_j = \infty$ for $j \neq s$ and $u_s = 0$
2. Put s in OPEN
3. Search OPEN for smallest $u_i + h_{i,r}$
4. For nodes j reached from i if $u_j > u_i + c_{ij}$ then $u_j = u_i + c_{ij}$
5. Put nodes j in OPEN and transfer i to CLOSED
6. Return to Step 3 until r is CLOSED

Spiess and Florian hyperpath algorithm

- *Hyperpath* is a bundle of potentially optimal paths
- Every link has both a cost and a service frequency
- Where there is choice within the hyperpath, allocation is proportional to service frequency (the *strategy*)
- Elemental path only added to hyperpath if the expected cost of travel is reduced

Hyperpath algorithm

1. Start at s and set $u_j = \infty$ for $j \neq$ destination, $u_s = 0$ and $F_i = 0$
2. Put s in OPEN
3. Search OPEN for smallest u_i
4. For nodes j reached from i if $u_j > u_i + c_{ij}$ then $u_j = (F_i u_i + f_{ij} c_{ij}) / (F_i + f_{ij})$, $F_i = F_i + f_{ij}$ and add link (i,j) to HP(r,s)
5. Put nodes j in OPEN and transfer i to CLOSED
6. Return to Step 3 until r is CLOSED

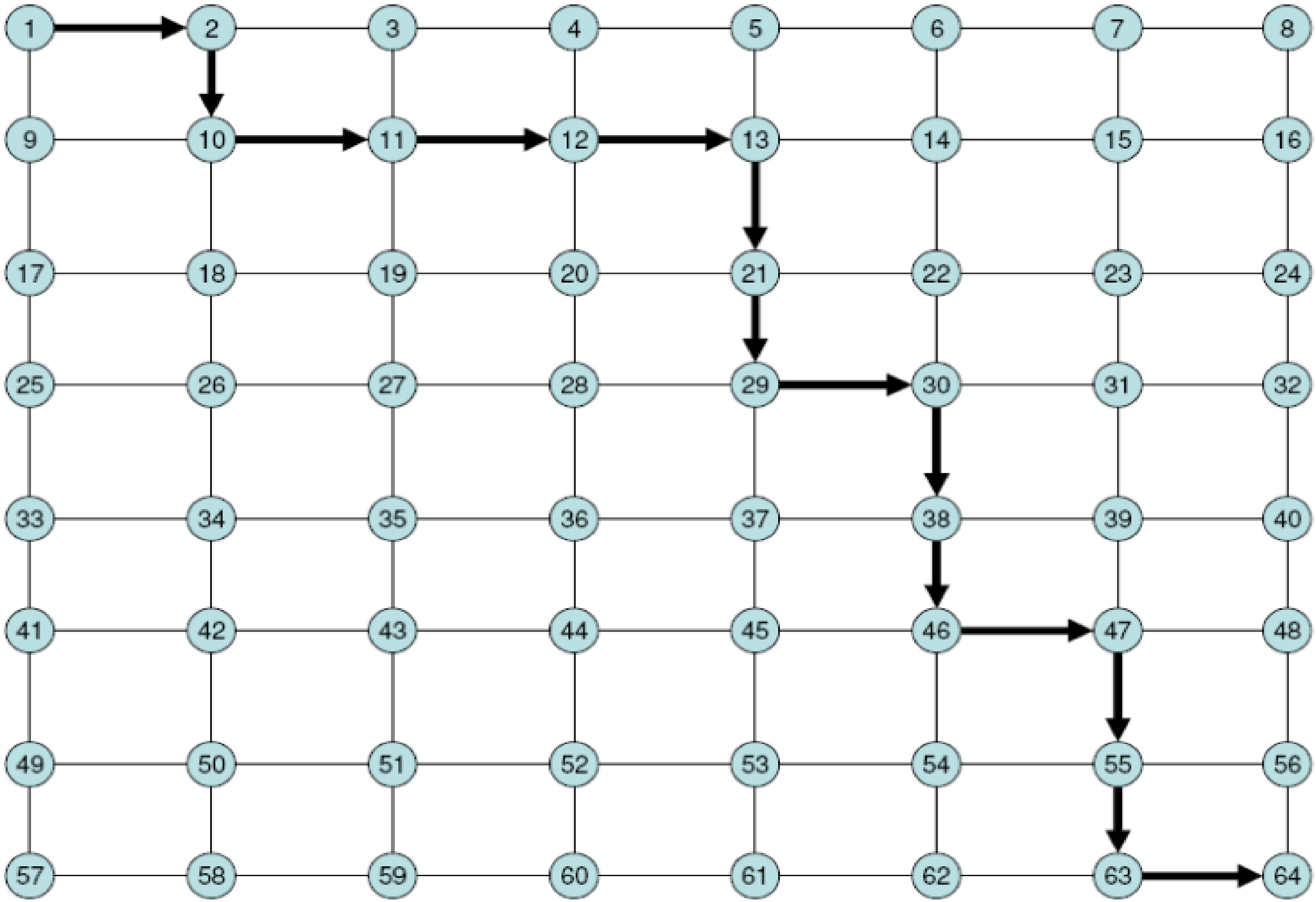
Reinterpreting the hyperpath algorithm

- Note: $1 \; / \; f_{ij}$ = link headway = max link delay = d_{ij}

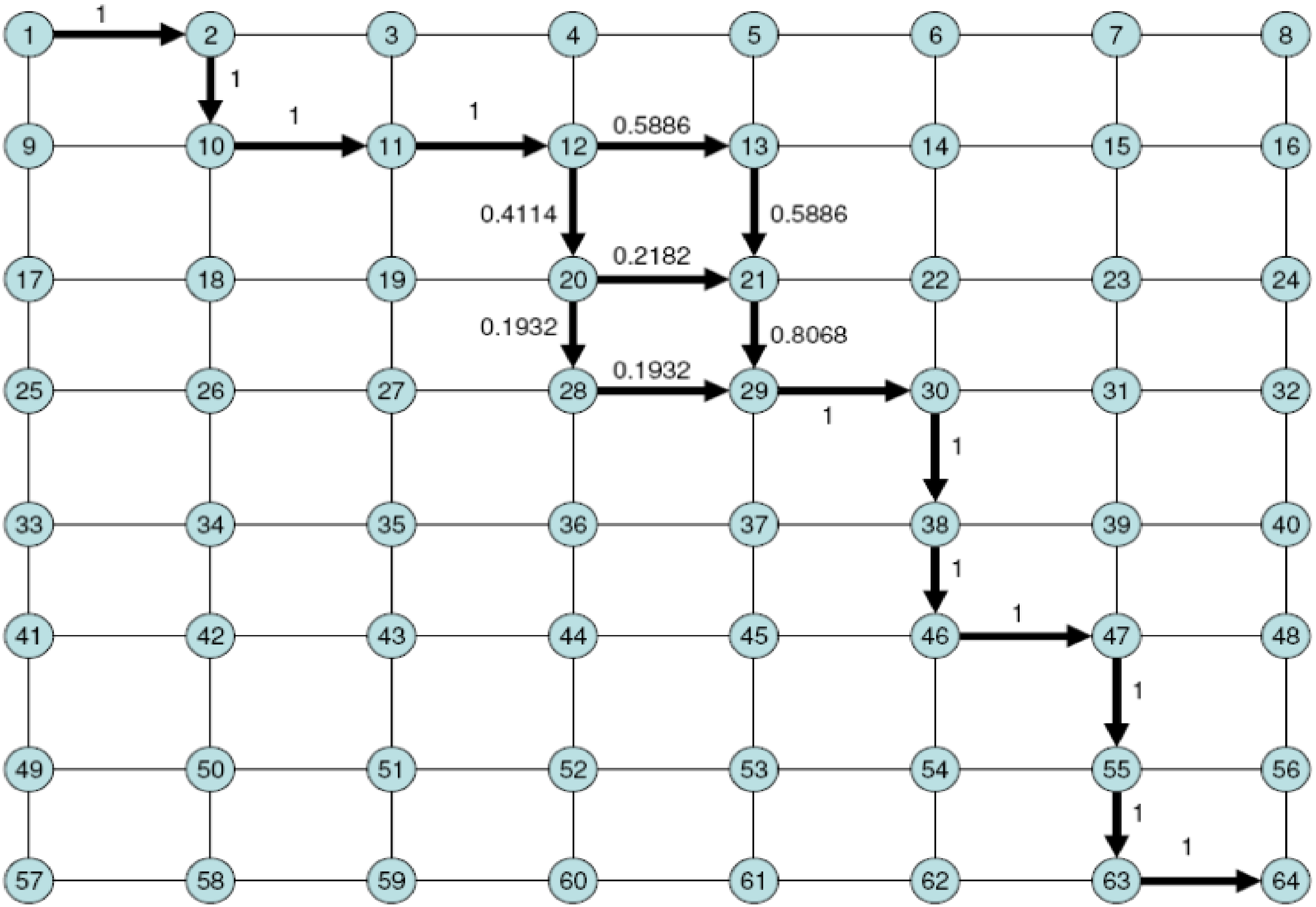
- Allocation: Minmax exposure to delay
  $\Rightarrow p_{ij} \, d_{ij} = p_{ik} \, d_{ik}$ if links (i,j) and (i,k) attractive
  $\Rightarrow p_{ij} \propto 1 \; / \; d_{ij} = f_{ij}$

- Attractive: Add link to hyperpath if "expected" travel time thereby reduced. Expected by whom? A risk averse traveller.

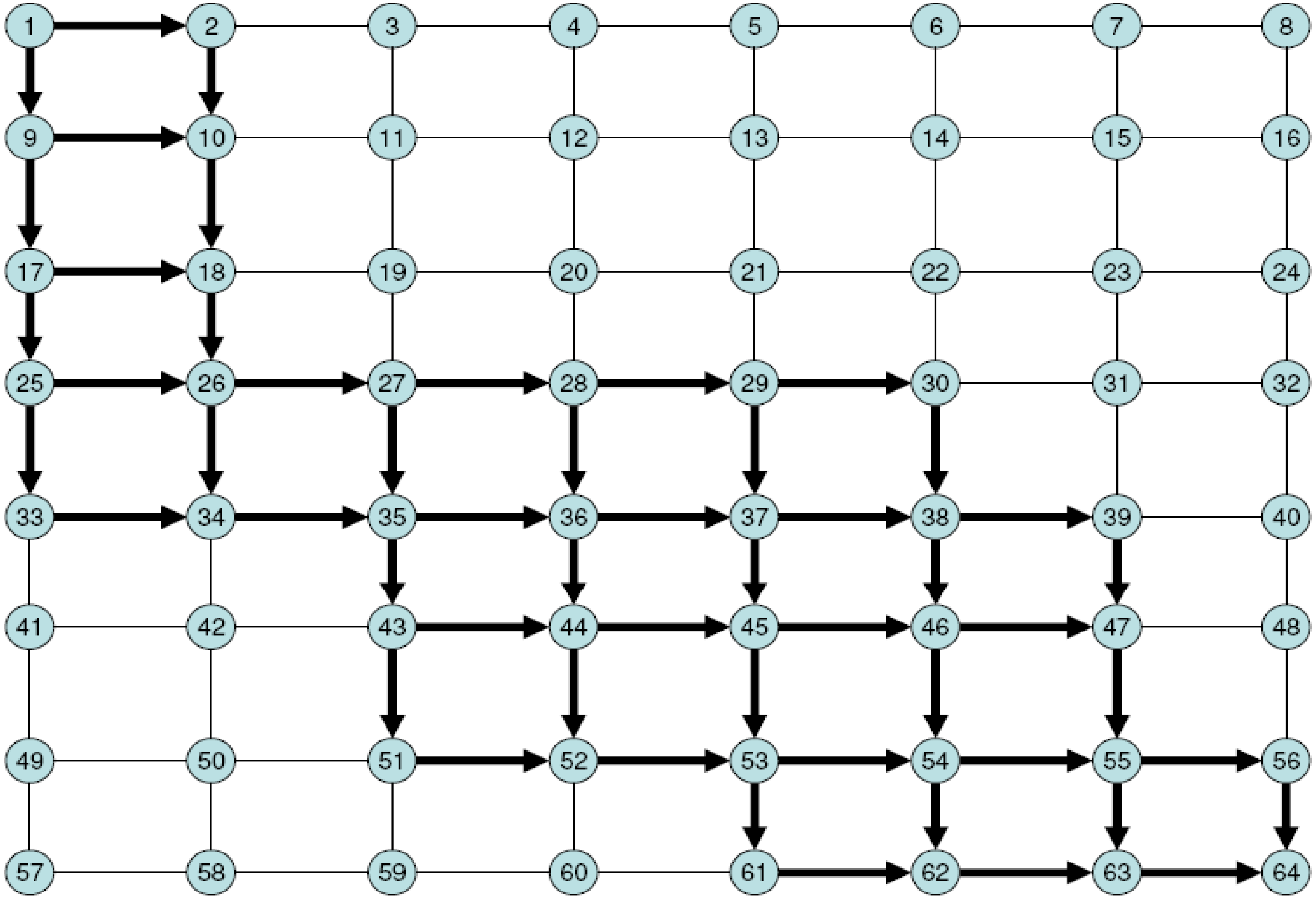
Singular hyperpath: No delay



Hyperpath: Med max link delays



Hyperpath: Large max link delays



H* algorithm

1. Start at destination and set $u_j = \infty$ for $j \neq s$, $u_s = 0$ and $F_i = 0$
2. Put s in OPEN
3. Search OPEN for smallest $u_i + h_{i,r}$
4. For nodes j reached from i if $u_j > u_i + c_{ij}$ then $u_j = (F_i\, u_i + f_{ij}\, c_{ij}) / (F_i + f_{ij})$, $F_i = F_i + f_{ij}$ and add link (i,j) to HP(r,s)
5. Put nodes j in OPEN and transfer i to CLOSED
6. Return to Step 3 until r is CLOSED

Time-dependent hyperpaths

- Reverse direction of search
- Requires FIFO

Transportation Research Part A 46 (2012) 790–800

Contents lists available at SciVerse ScienceDirect

Transportation Research Part A

journal homepage: www.elsevier.com/locate/tra

ELSEVIER

TRANSPORTATION RESEARCH

Time-dependent Hyperstar algorithm for robust vehicle navigation

Michael G.H. Bell [a,*], Valentina Trozzi [a], Solmaz Haji Hosseinloo [a], Guido Gentile [b], Achille Fonzone [a]

[a] *Centre for Transport Studies, Imperial College London, United Kingdom*
[b] *Dipartimento di Idraulica Trasporti e Strade, Sapienza Università di Roma, Italy*

Conclusions

- The demon game approach offers interesting solutions to both LPHC and HPLC problems

- Efficient solution algorithms exist for both types of problem

Imperial College London

THANK YOU