Making decisions in hazardous transport networks
Introduction

- **Aim**
  - Reliable transport in uncertain networks

- **Approach**
  - Game theory: Demon(s) try to disrupt trips
    - Single demon: Low probability – High consequence (LPHC)
    - Multiple demons: High probability – Low Consequence (HPLC)

- **Questions**
  - Where will demon(s) strike? Critical links
  - How to reduce the risk? Strategy

- **Solution**
  - LPHC: Olympic Route Network
  - HPLC: Vehicle navigation
PART 1 Introduction to the approach
- Uncertainty and risk
- Game theory

PART 2 Example: Olympic route network
- Single demon game
- Benefits from routing strategy
- Benefits from defence strategy

PART 3 Example: Vehicle navigation
- Multiple demon game
- Hyperstar algorithm
- Time–dependent vehicle navigation
Transport risk factors

Risk = incident probability \times incident impact

low probability \quad \text{high impact}

Community protest
- Terrorist attack
- Road accident
- Weather
- Traffic levels
- Time of the day
- Route

Residents
- Other travellers
- Environment
- Package quality
- Amount of waste
- Frequency of dispatches
- Time of the day
- Route
- Location – Allocation

Uncertainty about incident probability...

...so focus on consequence minimisation!

scope for further risk reduction
Example: disposal site and source allocation
Example: Combined routing and scheduling
PART 1
Research background

Uncertainty and Game Theory
Reliability – Vulnerability – Risk

- Security = acceptable level of risk
- Risk = potential loss
- Risk = hazard/threat \times vulnerability

\[ \text{EXTERNAL} \quad \text{INTERNAL} \]

- Vulnerability = inability to avoid potential harm
- Reliability = stability in the quality of service
Reliability vs Vulnerability

Costs

Cost of disruptions

£2m

£1m

Cost of countermeasures

overspending

Reliability

Vulnerability

ARUP
Decisions under uncertainty

Risk Averse Gambler

Probability density

Investment + Consequence

Total Cost

5%

30%

ARUP

Imperial College London
### Risk Aversion and Game Theory

#### Demon

- What to attack?

<table>
<thead>
<tr>
<th>£</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>12</td>
<td>-1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>T2</td>
<td>5</td>
<td>1</td>
<td>7</td>
<td>-20</td>
</tr>
<tr>
<td>T3</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>T4</td>
<td>-16</td>
<td>0</td>
<td>0</td>
<td>6</td>
</tr>
</tbody>
</table>

#### Gamblers

- Which route to take?

<table>
<thead>
<tr>
<th>£</th>
<th>MAX</th>
<th>MIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>12</td>
<td>-1</td>
</tr>
<tr>
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<tr>
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<td>4</td>
<td>2</td>
</tr>
<tr>
<td>T4</td>
<td>6</td>
<td>-16</td>
</tr>
</tbody>
</table>

#### Dispatcher

- Which route to select?

<table>
<thead>
<tr>
<th>£</th>
<th>MAX</th>
<th>MIN</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>T4</td>
<td>6</td>
<td>-16</td>
</tr>
</tbody>
</table>
How the game works? – Round 1

Disruption = increase in cost
How the game works? – Round 3
How the game works? - Equilibrium

Bar width = probability value

24%
76%
At the solution

- **Routes used**
  - Only routes attractive to the dispatcher are generated
  - Routes with minimum expected cost
  - Link use probabilities
    \[ \rightarrow \text{Safest path choice frequency} \]

- **Links attacked**
  - Only links attractive to the demon are attacked
  - Links with maximum expected loss
  - Only links with non-zero link use probability
  - Link failure probabilities
    \[ \rightarrow \text{Critical links} \]
PART 2
Application to Olympic routes
Routing & Defence Strategies
Transport game applied to ORN
Analysis of the ORN network

- **Single routing**
  - Without disruption
  - With disruption
    - minor $k=2$
    - major $k=1,000,000$

- **Multiple routing**
  - Without disruption
  - With disruption
    - minor $k=2$
    - major $k=1,000,000$

- **Multiple routing with active defence**
  - With disruption
    - major $k=1,000,000$

Potential losses

Potential benefits
Shortest path

Greenwich to Victoria Park
Disruption Cost = 1 x 10
Iterations: 1
PROTECTION TYPE: None

Solution cost: 727

Cost 727 sec
Single routing + major disruption

Greenwich to Victoria Park
Disruption Cost = 1 000 000 x 10
Iterations: 1
PROTECTION TYPE: None

Solution cost:
120 000 608

Cost 727 sec

Cost 120m sec
Multiple routing + major disruption

Disruption Cost = 1 000 000 x 10
Iterations: 500
PROTECTION TYPE: None

Solution cost:
24 130 840

Cost 727 sec
Cost 120m sec
Cost 24m sec

Saving 80%
## Major Disruption

<table>
<thead>
<tr>
<th>Total Cost (sec)</th>
<th>Does not Happen</th>
<th>Does Happen</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Single route</td>
<td>727</td>
<td>120 m</td>
</tr>
<tr>
<td>B Optimal routes</td>
<td>1102</td>
<td>24 m</td>
</tr>
</tbody>
</table>
Significant benefits from multiple routing at a relative low cost

Multiple routing mitigates the risk of a serious disruption

Routes with least expected costs are generated

Number of routes depends on the size of potential losses
Anticipated defence

- Cost 24m sec
- Cost 15m sec

Greenwich to Victoria Park
Disruption Cost = 1 000 000 x t0
Iterations: 500
PROTECTION TYPE: 3

Solution cost:
15 131 901
Transport games with defence

Considered defence types that:

- Visible
  - Links are not equally attractive to an attacker
  - An attack on some links has more severe consequences than
  - Critical links can be identified

...what is therefore optimal defence plan?
## Anticipated defence – path choice

<table>
<thead>
<tr>
<th></th>
<th>Blackwall Tunnel</th>
<th>Rotherhithe Tunnel</th>
<th>Tower Bridge</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DEFENCE</strong></td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
</tr>
<tr>
<td><strong>Link Use</strong></td>
<td>21%</td>
<td>14%</td>
<td>18%</td>
</tr>
<tr>
<td><strong>Link Attack</strong></td>
<td>20%</td>
<td>8%</td>
<td>17%</td>
</tr>
<tr>
<td><strong>Link Defence</strong></td>
<td>–</td>
<td>0%</td>
<td>–</td>
</tr>
<tr>
<td>Cost [million sec]</td>
<td>Defence type</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------</td>
<td>--------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Routing only</td>
<td>Visible</td>
<td>Invisible</td>
</tr>
<tr>
<td>Solution Cost</td>
<td>24</td>
<td>17</td>
<td>10</td>
</tr>
<tr>
<td>Benefit</td>
<td>–</td>
<td>7</td>
<td>14</td>
</tr>
<tr>
<td>% of the SC</td>
<td>–</td>
<td>30%</td>
<td>58%</td>
</tr>
</tbody>
</table>
Defence influences the optimal routing

Invisible defence yields max benefits

It is most beneficial to protect river crossings, in particular Tower Bridge.

Even if only one link is protected, the expected cost can be significantly reduced
## Application of the method

### Strategic
- Find critical links
- Estimate costs of various scenarios
- Establish optimal routing and defence strategies

### Operational
- Check what happens if some links are no longer available
- Produce contingency routes updated according to road conditions

### Navigation
- Produce individual routing plans for drivers
- Real time update using on-line traffic information
Refinements

- Flow dependent link costs
- Joint examination of multiple OD
- Link failure affecting both directions
- Attack and defence of multiple links
- Budget constraints
- Deceptive strategies
- Dynamic effects
Conclusions

- Multiple routing is a rational measure to distribute risk
- Potential for application
- Optimal routing & defence strategies bring significant quantifiable benefits
PART 3

Application to vehicle navigation

Strategic & Operational Planning and Navigation
Introduction

- LPHC implies one demon
- HPLC implies multiple demons

HPLC:

- Place a demon at every node
- Solve by a version of the Spiess and Florian hyperpath algorithm
- Accelerated by node potentials
Assumptions

- Every link $a \in A$ has a cost of use $c_a$ under normal operating conditions.
- There is an additional cost of use $d_a$ if the link is congested.
- Worst case: On exiting any node $i \in N$, one link is degraded.
- Seek link use probabilities that minimise expect travel cost subject to worst case link congestion probabilities.
Demon games and the minmax exposure principle

- Every node has a demon with the ability to fail one outgoing link.

- Consider a zero sum game, where each demon can select one outgoing link $a$ to impose $d_a$ and the dispatcher seeks a least cost route with respect to $c_a$ and expectation of $d_a$ (Schmoecker et al., 2009).

- Find the mixed strategy Nash equilibrium by:

$$\min_p \left( \sum_{a \in A} c_a p_{as} + \max_q \sum_{a \in A} q_{as} d_a p_{as} \right)$$
Hypertrees and hyperpaths

- Probability $q_{as}^*$ measures link criticality

- Links with probability $p_{as}^* > 0$ define the **hypertree** to $s$

- $p_{as}^* > 0 \iff q_{as}^* > 0$ and $p_{as}^* = 1 \iff q_{as}^* = 1$

- The hyperpath cost is

$$u_{rs} = \sum_{a \in HP(r,s)} c_a p_{as}^* + \sum_{a \in HP(r,s)} q_{as}^* d_a p_{as}^*$$
Hyperpath generation

\[ \text{Min}_{p,w} \sum_{s \in S} \left( \sum_{a \in A} c_a p_{as} + \sum_{i \in I} w_{is} \right) \]

subject to

\[ \sum_{a \in A_i^+} p_{as} - \sum_{a \in A_i^-} p_{as} = g_{is}, \forall i \in I, s \in S \]

\[ w_{is} \geq p_{as} d_a, \forall a \in A_i^-, i \in I, s \in S \]

\[ p_{as} \geq 0, \forall a \in A, r \in R, s \in S \]
Dijkstra’s algorithm

1. Start at $s$ and set $u_j = \infty$ for $j \neq s$ and $u_s = 0$

2. Put $s$ in OPEN

3. Search OPEN for smallest $u_i$

4. For nodes $j$ reached from $i$ if $u_j > u_i + c_{ij}$ then $u_j = u_i + c_{ij}$

5. Put nodes $j$ in OPEN and transfer $i$ to CLOSED

6. Return to Step 3 until $r$ in CLOSED
A* algorithm

1. Start at \( s \) and set \( u_j = \infty \) for \( j \neq s \) and \( u_s = 0 \)

2. Put \( s \) in OPEN

3. Search OPEN for smallest \( u_i + h_{i,r} \)

4. For nodes \( j \) reached from \( i \) if \( u_j > u_i + c_{ij} \) then \( u_j = u_i + c_{ij} \)

5. Put nodes \( j \) in OPEN and transfer \( i \) to CLOSED

6. Return to Step 3 until \( r \) is CLOSED
Spiess and Florian hyperpath algorithm

- *Hyperpath* is a bundle of potentially optimal paths.
- Every link has both a cost and a service frequency.
- Where there is choice within the hyperpath, allocation is proportional to service frequency (the *strategy*).
- Elemental path only added to hyperpath if the expected cost of travel is reduced.
Hyperpath algorithm

1. Start at $s$ and set $u_j = \infty$ for $j \neq$ destination, $u_s = 0$ and $F_i = 0$

2. Put $s$ in OPEN

3. Search OPEN for smallest $u_i$

4. For nodes $j$ reached from $i$ if $u_j > u_i + c_{ij}$ then $u_j = (F_i u_i + f_{ij} c_{ij}) / (F_i + f_{ij})$, $F_i = F_i + f_{ij}$ and add link $(i, j)$ to HP$(r, s)$

5. Put nodes $j$ in OPEN and transfer $i$ to CLOSED

6. Return to Step 3 until $r$ is CLOSED
Reinterpreting the hyperpath algorithm

- Note: $1 / f_{ij} = \text{link headway} = \text{max link delay} = d_{ij}$

- Allocation: Minmax exposure to delay
  \[ p_{ij} \ d_{ij} = p_{ik} \ d_{ik} \] if links $(i,j)$ and $(i,k)$ attractive
  \[ p_{ij} \propto 1 / d_{ij} = f_{ij} \]

- Attractive: Add link to hyperpath if “expected” travel time thereby reduced. Expected by whom? A risk averse traveller.
Singular hyperpath: No delay
Hyperpath: Med max link delays
Hyperpath: Large max link delays
**H* algorithm**

1. Start at destination and set $u_j = \infty$ for $j \neq s$, $u_s = 0$ and $F_i = 0$

2. Put $s$ in OPEN

3. Search OPEN for smallest $u_i + h_{i,r}$

4. For nodes $j$ reached from $i$ if $u_j > u_i + c_{ij}$ then $u_j = (F_i u_i + f_{ij} c_{ij}) / (F_i + f_{ij})$, $F_i = F_i + f_{ij}$ and add link $(i,j)$ to HP($r,s$)

5. Put nodes $j$ in OPEN and transfer $i$ to CLOSED

6. Return to Step 3 until $r$ is CLOSED
Time-dependent hyperpaths

- Reverse direction of search
- Requires FIFO

Time-dependent Hyperstar algorithm for robust vehicle navigation

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\textsuperscript{b} Dipartimento di Idraulica Trasporti e Strade, Sapienza Università di Roma, Italy
Conclusions

- The demon game approach offers interesting solutions to both LPHC and HPLC problems
- Efficient solution algorithms exist for both types of problem
THANK YOU